8 Calculator Skills

A Brackets and Negatives

A scientific calculator 'knows' the order of operations, but it cannot 'see' the hidden brackets.

Examples : $20 - 4 \times {}^{-}3$ can be keyed in as . . .

$\boxed{20}\ \boxed{-}\ \boxed{4}\ \boxed{\times}\ \boxed{(-)}\ \boxed{3}\ \boxed{=}$ answer 32,

but $\dfrac{23 - {}^{-}2}{8 + {}^{-}3}$ must be keyed in as . . .

$\boxed{(}\ \boxed{23}\ \boxed{-}\ \boxed{(-)}\ \boxed{2}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{8}\ \boxed{+}\ \boxed{(-)}\ \boxed{3}\ \boxed{)}\ \boxed{=}$

answer 5.

1 Calculate these in one run.

a) $2 \times {}^{-}5 - {}^{-}10 \times 2$ ....

b) $(16 - 34)\,(100 - 15)$ ....

c) $39 - 3\,(15 - 8) + 4$ ....

2 Key in brackets where needed to calculate these in one run.

a) $\dfrac{85 - 17}{{}^{-}4}$ ....

b) $\dfrac{25}{4} \times \dfrac{8}{2 - {}^{-}3}$ ....

c) $\dfrac{4\,(18 + {}^{-}6)}{6 \times 2.5}$ ....

B Powers and Square Roots

Use $\boxed{\wedge}$ for powers and $\boxed{\sqrt{\ }}$ for square roots.

Examples : 5^8 is keyed in as $\boxed{5}\ \boxed{\wedge}\ \boxed{8}\ \boxed{=}$, answer 390 625

$\sqrt{30 + 34}$ is keyed in as . . .

$\boxed{\sqrt{\ }}\ \boxed{(}\ \boxed{30}\ \boxed{+}\ \boxed{34}\ \boxed{)}\ \boxed{=}$, answer 8

1 Calculate these.

a) 2^{-2} ....

b) $({}^{-}2 + 5)^3$ ....

c) $\sqrt{4^8}$ ....

d) $\frac{1}{2}\sqrt{5} \times \sqrt{20}$ ....

2 True or False? Write T or F .

a) ${}^{-}3^2 = ({}^{-}3)^2$ [ ]

b) $\sqrt{9 + 16} = \sqrt{9} + \sqrt{16}$ [ ]

c) $\sqrt{9 \times 16} = \sqrt{9} \times \sqrt{16}$ [ ]

d) $2^{-3} = \dfrac{1}{2^3}$ [ ]

C Fractions

Examples : a) Calculate $\frac{2}{3}$ of \$9.75.

b) Write the mixed number $2\frac{4}{5}$ as an improper fraction.

Working : a) $\boxed{2}\ \boxed{a^b\!/\!c}\ \boxed{3}\ \boxed{\times}\ \boxed{9.75}\ \boxed{=}$ answer \$6.50

b) $\boxed{2}\ \boxed{a^b\!/\!c}\ \boxed{4}\ \boxed{a^b\!/\!c}\ \boxed{5}\ \boxed{=}\ \boxed{d\!/\!c}$ answer $\frac{14}{5}$

1 Use the $\boxed{a^b\!/\!c}$ key to calculate these.

a) $\frac{2}{3} + \frac{1}{4}$ .... b) $2\frac{1}{2} - 1\frac{4}{9}$ ....

c) $\frac{5}{8} \div \frac{1}{2}$ .... d) $3\frac{2}{3} \times 6$ ....

e) $\frac{3}{8}$ of \$15.60 ....

f) $\frac{1}{2}\,(2\frac{1}{4} + 3\frac{5}{8})$ ....

2 Use $\boxed{a^b\!/\!c}$ and $\boxed{d\!/\!c}$ for these.

a) Write $\frac{43}{8}$ as a mixed number. ....

b) Write $3\frac{1}{12}$ as an improper fraction. ....

c) Write $\frac{18}{30}$ with lowest numbers. ....

D Investigating the $\boxed{\text{EXP}}$ Key

1 Write down the result of these.

a) $\boxed{2}\ \boxed{\text{EXP}}\ \boxed{3}\ \boxed{=}$ ....

b) $\boxed{2}\ \boxed{\text{EXP}}\ \boxed{5}\ \boxed{=}$ ....

c) $\boxed{6.5}\ \boxed{\text{EXP}}\ \boxed{3}\ \boxed{=}$ ....

d) $\boxed{6.5}\ \boxed{\text{EXP}}\ \boxed{5}\ \boxed{=}$ ....

2 Try other combinations e.g.

a) $\boxed{2}\ \boxed{\text{EXP}}\ \boxed{0}\ \boxed{=}$ ....

b) $\boxed{2}\ \boxed{\text{EXP}}\ \boxed{{}^{-}1}\ \boxed{=}$ ....

c) $\boxed{2}\ \boxed{\text{EXP}}\ \boxed{3.5}\ \boxed{=}$ ....

d) $\boxed{\ \ }\ \boxed{\text{EXP}}\ \boxed{\ \ }\ \boxed{=}$ ....

3 Describe what $\boxed{a}\ \boxed{\text{EXP}}\ \boxed{b}\ \boxed{=}$ does.

....

....

....

A On the Number Line

$a = b$ means a is equal to b
$a < b$ means a is less than b
$a \le b$ means a is less than or equal to b
$a > b$ means a is greater than b
$a \ge b$ means a is greater than or equal to b

Number sentences with $=$ are called equations.
Number sentences with $<, >, \le, \ge$ are called inequations.
Note : $a < x < b$ means $x > a$ and $x < b$ simultaneously.

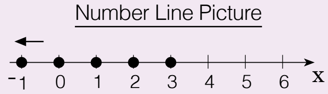
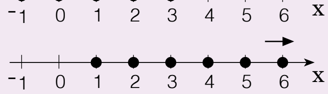
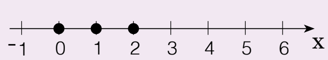
1 Are these number sentences true or false? Write **T** or **F** .

a) $3.42 > 3.24$ ☐ b) $1\frac{3}{4} = 1.75$ ☐
c) $\frac{9}{4} < 2$ ☐ d) $^-6 > 4$ ☐
e) $2.6 = 2.06$ ☐ f) $0.3 < \frac{1}{3}$ ☐
g) $^-3 < 0 < 2$ ☐ h) $^-3 > 0 > 2$ ☐

2 True or False?

a) $2 < x$ is the same as $x > 2$ ☐
b) $2 < x < 5$ is the same as x is between 2 and 5 ☐

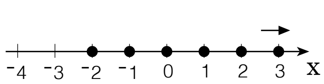
Inequations can be shown on a number line.
Examples (we assume that x is an integer) :

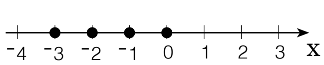
Inequation	Number Line Picture
$x < 4$	
$x \ge 1$	
$0 \le x < 3$	

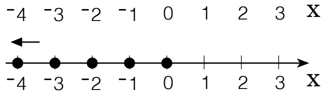
3 Show these inequations on the number line.

a) $x \le 2$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 x
b) $x > {}^-1$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 x
c) $^-2 < x \le 2$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 x

4 Write inequations for these pictures.

a)  ....

b)  ....

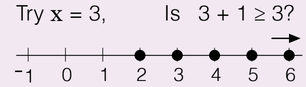
c)  ....

B Solving Inequations

Solving inequations can be done on the number line using guess and check.

Examples : For what integer values of x, is $x + 1 \ge 3$?

Working :			
	Try $x = {}^-1$,	Is $^-1 + 1 \ge 3$?	No, it is less
	Try $x = 0$,	Is $0 + 1 \ge 3$?	No, it is less
	Try $x = 1$,	Is $1 + 1 \ge 3$?	No, it is less
	Try $x = 2$,	Is $2 + 1 \ge 3$?	Yes, it is equal
	Try $x = 3$,	Is $3 + 1 \ge 3$?	Yes, it is larger

Picture : 

Solution : $x \ge 2$

1 Solve these inequations in which x is an integer.

a) $x - 1 \le 0$ $^-3$ $^-2$ $^-1$ 0 1 2 3 x

Solution ....

b) $2x + 1 \ge 5$ $^-3$ $^-2$ $^-1$ 0 1 2 3 x

Solution ....

c) $^-3x > 3$ $^-3$ $^-2$ $^-1$ 0 1 2 3 x

Solution ....

2 Draw your own number lines to solve these.

a) $x - 2 < 1$ x

Solution ....

b) $\frac{x}{2} \ge 3$ x

Solution ....

c) $3x - 2 > 4$ x

Solution ....

d) $\frac{x - 1}{3} < 1$ x

Solution ....

e) $2 - x \ge 0$ x

Solution ....

f) $\frac{x}{4} + 1 \ge {}^-1$ x

Solution ....

48 Plotting Lines 1

A Graphing a Rule

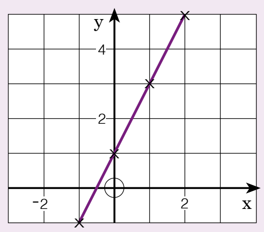
If we have a rule we can make a table.
If we have a table we can plot a graph.

Example : a) Make a table for the rule $y = 2x + 1$.
b) Plot the graph.

Working :

a)

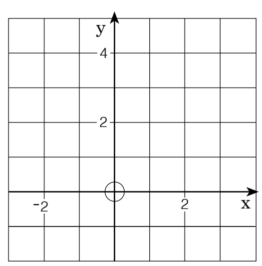
x	y (= 2 x x + 1)
$^-1$	$2 \times {}^-1 + 1 = {}^-1$
0	$2 \times 0 + 1 = 1$
1	$2 \times 1 + 1 = 3$
2	$2 \times 2 + 1 = 5$



b) Draw the graph by plotting the points ($^-1$, $^-1$), (0, 1), (1, 3), (2, 5), then joining them with a long straight line.

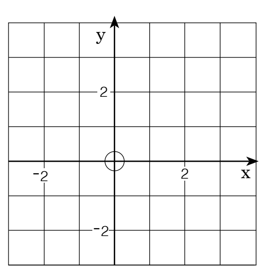
1 The rule is $y = x + 3$.
Complete the table and draw the graph.

x	y (= x + 3)
$^-1$	
0	
1	
2	



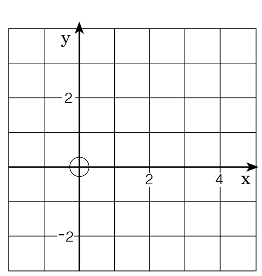
2 The rule is $y = {}^-x$ (this is the same as $y = {}^-1x$).
Complete the table and draw the graph.

x	y (= $^-1$ x x)
$^-1$	
0	
1	
2	



3 The rule is $y = 2x - 1$.
Complete the table and draw the graph.

x	y (= 2 x x − 1)



B Two Points is Enough

If the rule is of the form $y = mx + c$ then the graph will be a straight line (m and c can be integers, fractions or decimals).
To draw a line it is enough to find the position of just two points on this line. The plotting can be made easy by choosing the x-values conveniently.

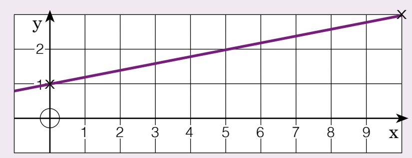
Example : Draw the graph of the rule $y = 0.2x + 1$

Working : Since the equation is of the form $y = mx + c$, the graph will be a line and two points are enough.

Choose x conveniently.

$x = 0$, $y = 0.2 \times 0 + 1 = 1$ (0, 1) easy to plot ✓
$x = 1$, $y = 0.2 \times 1 + 1 = 1.2$ (1, 1.2) not easy to plot ✗
$x = 10$, $y = 0.2 \times 10 + 1 = 3$ (10, 3) easy to plot ✓

Plot the points and draw the line.



Note : It is a good idea to plot a third point to make sure that the line is in the correct place.
$x = 5$, $y = 0.2 \times 5 + 1 = 2$, giving (5, 2) which is on the line.

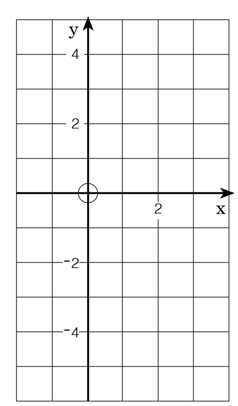
1 Plot two lines on the same grid.
For each line find 3 points choosing the x-value conveniently.

a) $y = 3x - 5$

(……, ……) (……, ……) (……, ……)

b) $y = {}^-2x + 3$

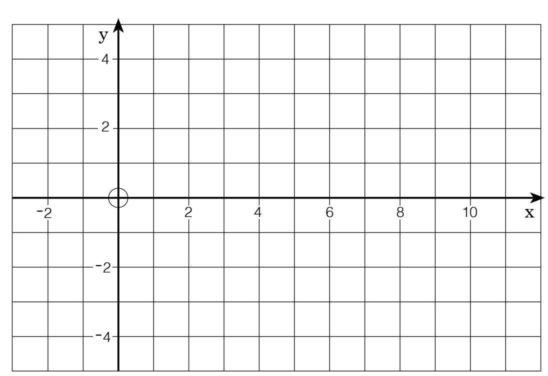
(……, ……) (……, ……) (……, ……)



2 Plot two lines on the same grid.

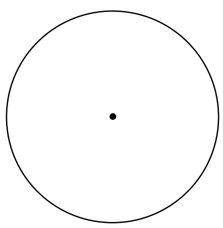
a) $y = 0.3x - 2$ (……, ……) (……, ……) (……, ……)

b) $y = {}^-0.4x + 3$ (……, ……) (……, ……) (……, ……)



A Parts of a Circle

Circumference is the word we use for the perimeter of a circle.
The centre of a circle is its midpoint. The radius is a straight line from the centre to any point on the circumference.
The diameter is a straight line going through the centre joining two points on the circumference.
The circumference of a circle is about 3 times as long as its diameter.

1


a) Label with C the centre of the circle.

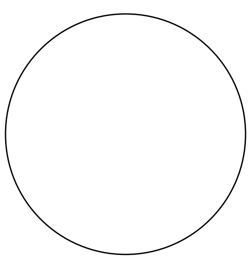
b) Draw and label with r the radius of the circle.

c) Measure the diameter of the circle.

……………………………………….

d) Estimate the circumference of the circle.

……………………………………………………………………...

2


It is not known where the centre of this circle is.

a) Explain how you would measure the length of the diameter.

……………………………………..

……………………………………..

……………………………………………………………………..

b) Measure the diameter ……………………………… cm.

c) Calculate the radius ……………………………… cm.

d) Estimate the circumference ……………………………… cm.

3 The circumference of this bicycle wheel is 165 cm.

a) Estimate its diameter.

……………………………………………….

b) Estimate its radius.

……………………………………………….

4 A forestry worker cuts all trees with a radius of 22 cm or more. He measures the circumference of 3 trees. Will he cut them?

Tree A has a circumference of 150 cm. ……………………..

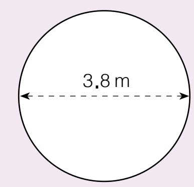
Tree B has a circumference of 125 cm. ……………………..

Tree C has a circumference of 140 cm. ……………………..

B Using a Formula

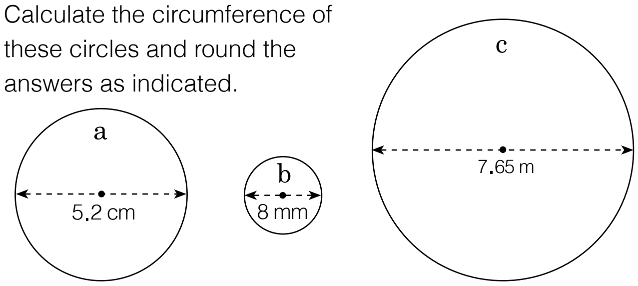
The exact circumference of a circle is found using the formula $C = \pi \times d$ where C is the circumference, d the diameter and $\pi = 3.141592654$. Fortunately π can be found on your calculator!
It is important to round your answer sensibly when using this formula. Round to the same number of decimal places as the measurements given in the problem.

Example :
With the formula $C = \pi \times 3.8$ the circumference of this circle is 11.93805208 metres.
Since the diameter was measured to 1 decimal place, the circumference is best rounded to 1 dp as well.



$C = 11.9|3805208 = 11.9$ m (1 dp)

1 Calculate the circumference of these circles and round the answers as indicated.



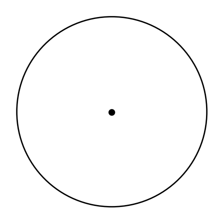
a) C = ……………………………………….. cm (1 dp)

b) C = ……………………………………….. mm (nearest mm)

c) C = ……………………………………….. m (2 dp)

2a) Measure and use the scale to work out the diameter in real life.

……………………………………….

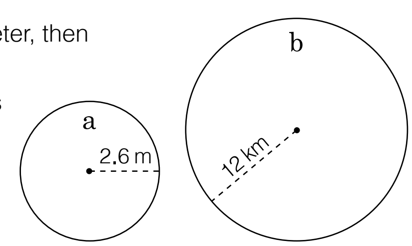


1 cm : 4 m

b) Calculate the circumference.

……………………………………….

3 Work out the diameter, then the circumference. Round the answers sensibly.



a) ……………………………………………………………………

……………………………………………………………………

b) ……………………………………………………………………

……………………………………………………………………

80 Similar Shapes 2

AS 1.7
Right-Angled Triangles

A Similar Triangles

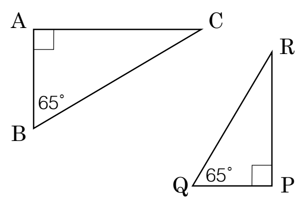
To check whether two right-angled triangles are similar, look for the following :

either : the two triangles have another angle size in common.

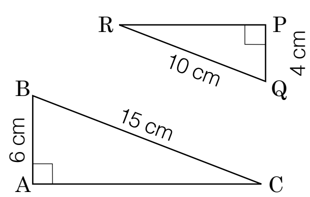
or : two sets of corresponding sides are scaled by the same factor.

1 Explain why triangles ABC and PQR must be similar.

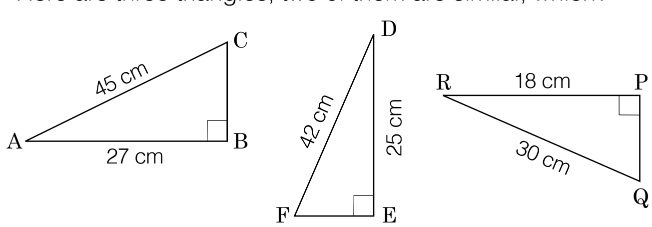
a)



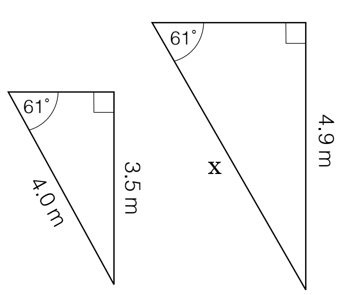
b)



2 Here are three triangles, two of them are similar, which?



3

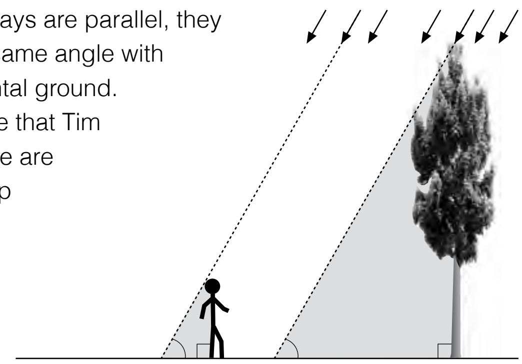


a) How do we know these triangles are similar?

b) Calculate the length of x.

B Real Life Applications

1 The sun's rays are parallel, they make the same angle with the horizontal ground. We assume that Tim and the tree are standing up straight.



Tim, with a height of 1.7 metres, casts a shadow of 0.9 m. The tree casts a shadow of 3.8 m. Explain how you could use this information to calculate the height of the tree.

2 Two catamarans have similar shapes, their masts are respectively 8 m and 10 m high. The mainsail of the smaller catamaran has an area of 14.5 m^2.

a) Calculate the scale factor k for the lengths of the masts.

b) Show how you work out the area of the main-sail of the larger catamaran.

3 If a photocopier is set at 70%, all lengths become 0.7 of the original lengths. A picture with an area of 240 cm^2 is copied at 70%. What is the area of the image?

A Perpendicular Lines

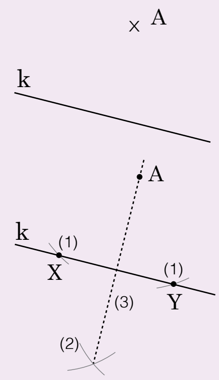
Constructing a line through A and perpendicular to k is done as follows :

Open your compasses far enough to reach from A just past line k.

(1) With the point in A, draw two arcs on k (X and Y).

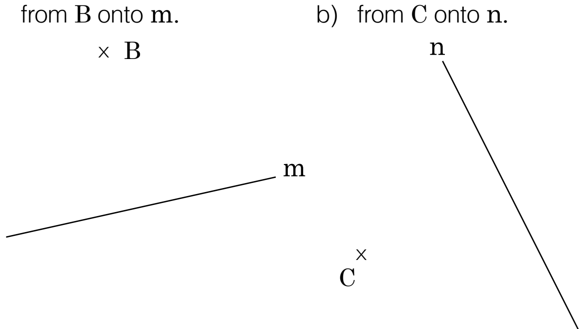
(2) Place the point in X and Y in turn and draw an arc-crossing.

(3) Join the arc-crossing with A to form the perpendicular line.

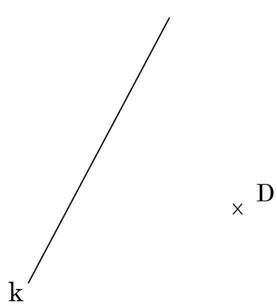


1 Construct perpendicular lines . . .

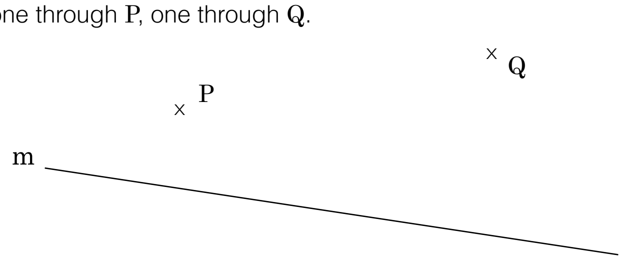
a) from B onto m. b) from C onto n.



c) from D onto k.

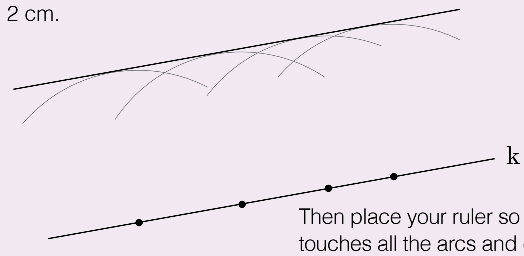


2 Construct two perpendicular lines onto m, one through P, one through Q.



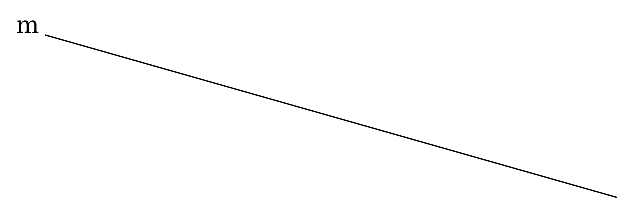
B Parallel Lines

Constructing a line parallel to line k at 2 cm distance can be done by drawing a series of arcs with a compass opening of 2 cm.



Then place your ruler so that it touches all the arcs and draw a line.

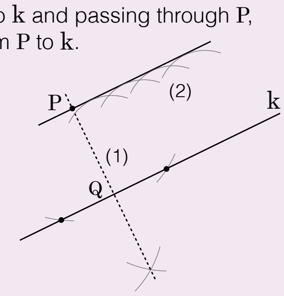
1 Draw a line parallel to m at 3 cm distance.



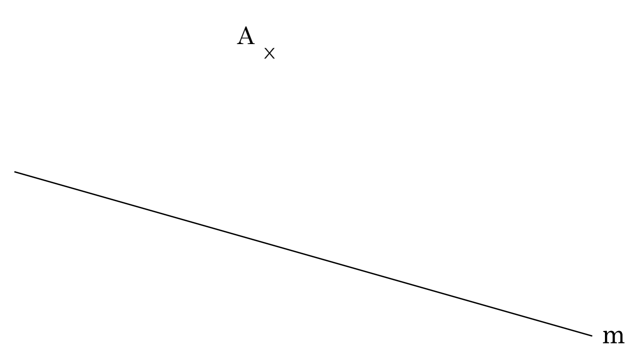
When constructing a line parallel to k and passing through P, you must first find the distance from P to k.

(1) Construct a perpendicular line like we did in Exercise A

(2) Distance PQ is the compass opening we need to draw the parallel line with a series of arcs.



2 Draw a line parallel to m, passing through A.



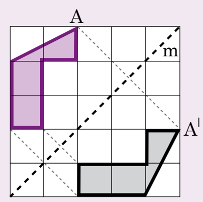
A Flip

A reflection flips a shape over to the other side of a line. That line is called the mirrorline. Original and image have the same distance to the mirrorline, also AA^I is perpendicular to m.

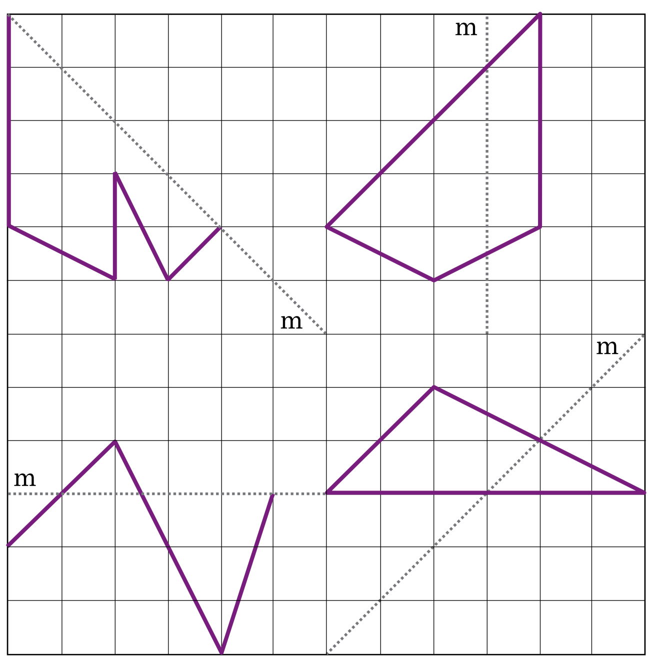
Example :

The grey shape is a reflection of the orange shape.

Notice that A and A^I have the same distance to the mirrorline m.

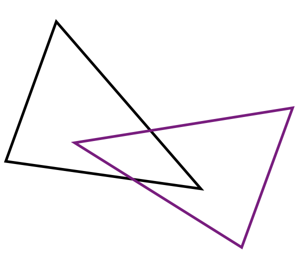


1 Draw the reflections of the shapes in the dotted mirrorlines.

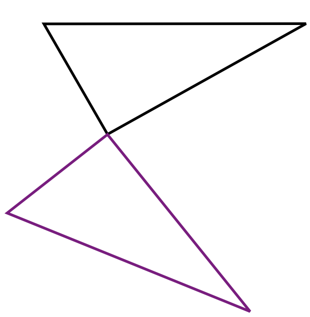


2 The purple triangle is reflected to give the black triangle. Draw the position of the mirrorline.

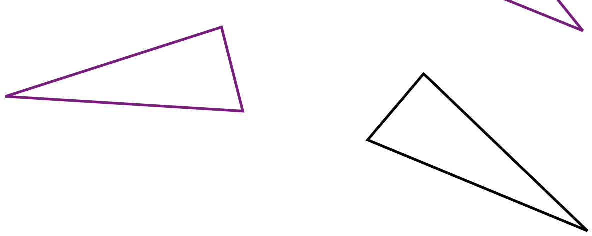
a)



b)

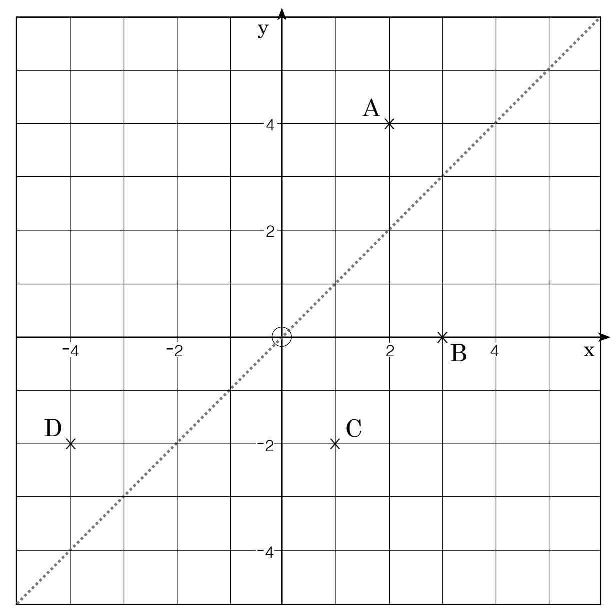


c)



B Reflection on the Cartesian Plan

1



Points A, B, C and D will be reflected in either the x-axis, or the y-axis or line m. Complete the table.

original point	appropriate	image point
A (2, 4)	x-axis	(………, ………)
B (3, 0)	x-axis	(………, ………)
C (1, $^-2$)	x-axis	(………, ………)
A (2, 4)	y-axis	(………, ………)
C (1, $^-2$)	y-axis	(………, ………)
D ($^-4$, $^-2$)	y-axis	(………, ………)
B (3, 0)	line m	(………, ………)
C (1, $^-2$)	line m	(………, ………)
D ($^-4$, $^-2$)	line m	(………, ………)

2a) Point $P(^-6,\ 10)$ is reflected in the y-axis.

What are the coordinates of P^I? (………, ………)

b) Now P^I is reflected in the line m.

What are the coordinates of P^{II}? (………, ………)

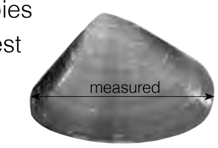
c) What if $P(^-6,\ 10)$ was first reflected in line m, and then the image was reflected in the y-axis.
Would we get the same final result?

…………………………………………………………………………

…………………………………………………………………………

A Shell Sizes

1 Tim has two favourite places where he gathers pippies. He reckons site A has larger pippies than site B. Tim investigates his assertion by collecting 30 pippies from each site. He measured them to the nearest millimetre, at the widest part of the shell.



Site A					Site B				
57	64	65	71	54	50	56	49	68	60
46	74	58	53	35	62	57	47	65	47
48	62	76	57	58	34	59	63	51	43
72	43	44	60	38	40	37	54	59	58
62	59	41	57	47	53	45	39	47	56
67	38	44	55	54	41	50	43	58	44

a) Draw a back-to-back stem and leaf plot for the data. (Use scrap paper first, then order the leaves and copy the result into the space provided.)

Shell Sizes (mm)

site A

site B

b) Calculate summary statistics

	site A	site B
mean		
median		
LQ		
UQ		
IQR		
range		

c) Draw box and whisker plots.

d) Write a few sentences comparing pippies from the two sites. Use the correct language.

....

....

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....

e) Write a conclusion. Can we accept Tim's assertion? Give a reason.

....

....

....

....

....

A Bon Voyage

A scattergraph is used to investigate the relationship between two features of an item. The graph is a series of plotted points. If the plotted points form a more or less straight strip then the scattergraph shows a linear relationship.

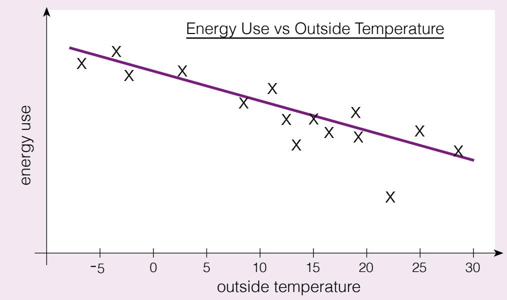
In that case a line of best fit (also called a trend line) can be drawn through the strip. This line can be used to make estimates. The closer the points are to the trend line, the stronger the relationship. If the line has a positive gradient (pointing upward), we say the relationship is positive, i.e. *'as the x variable increases so does the y variable.'* If the line has a negative gradient (pointing downward), we say the relationship is negative, i.e. *'as the x variable increases, the y variable decreases.'*

When asked to discuss features of the scatter plot, look for clusters of datapoints, look for outliers (a point which is notably outside the pattern), and discuss the spread or variation of the data.

Example :

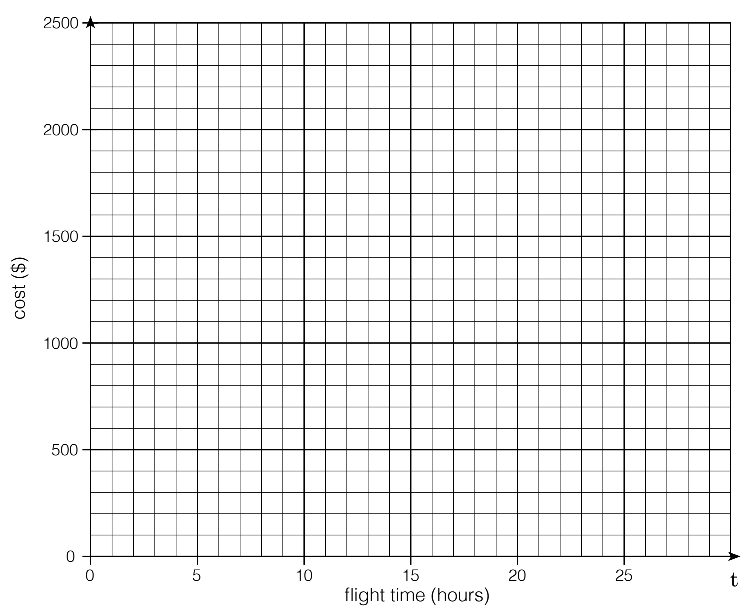
This scatter plot shows energy usage in households against outside temperature. Since the line is going down we could make the following comments :

- *There is a strong negative linear relationship, the higher the outside temperature the lower the energy use.*
- *There is more variation in energy usage when temperatures are moderate.*
- *At lower temperatures the points are closer to the trend line.*
- *There is one outlier, at 22°C we expect a higher energy use than shown.*



1 The table below shows flight duration and airfare for flights departing from Auckland (prices August 2010).

Destination	Duration	Return Fare
Adelaide	5h 30m	$ 809
Bangkok	13h 30m	$ 981
Cairns	6h 30m	$ 836
Christchurch	1h 20m	$ 171
Johannesburg	17h 40m	$1864
London	26h 45m	$1979
Los Angeles	13h 20m	$1496
Sydney	3h 30m	$ 424
Tokyo	13h 20m	$1130
Vancouver	15h 20m	$1564



a) Complete the scatter plot.

b) Draw a line of best fit through the points.

c) Describe the relationship between flight duration and airfare. ....

....

d) There are 3 flights with a duration of about $13\frac{1}{2}$ hours. Describe the variation in return fares for these flights.

....

e) i) List a destination which seems to be an outlier. ....

ii) Which destinations have high airfares? ....

iii) Can you think of a reason for these occurrences? ....

....

....

f) It takes 10 hours and 45 minutes to fly from Auckland to Hong Kong. Estimate the airfare. ....

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