

A Work Out the Original Amount

If we know a percentage of an amount, then calculating the whole amount can be done by solving an equation.

Example : If 35% of an amount equals \$6.86, how much was the original amount?

Working : Let the original amount be x , then 0.35 of $x = 6.86$
 or $0.35 \times x = 6.86$

$$x = \frac{6.86}{0.35} = \$19.60$$

1a) 18% of a volume equals 459 mL. Calculate the original volume.

.....

b) 85% of a mass equals 64.6 kg. Calculate the original mass.

.....

c) 120% of an amount equals \$75 000. Calculate the original amount.

.....

2a) 40% of a bag of cashew nuts weighs 260 g. How much did the bag full of nuts weigh?

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b) 21% of Mrs Pachett's wages go to tax. If Mrs Pachett pays \$274.60 tax, how much are her wages?

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C More Practice

1 After months of hard training Danelle increased her best shot-put distance by $\frac{1}{10}$. Her best distance is now 18.48 m. What was her best distance before training?

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2 85% of kiwi chicks hatched in an unprotected area were killed by dogs and ferrets. There were twelve surviving chicks in this area. How many chicks hatched?

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3a) Kylie's car decreases in value by 18% each year. This year Kylie's car has a value of \$ x . Write down an expression for its value next year.

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b) Write down an expression for its value last year.

B Writing an Equation

Example : After a wage increase of 3% an employee now earns \$762.20 a week. What were her wages before the increase?

Explanation : Let w be the original wages.
 New wages = $w + 3\%$ of w
 $= 1 \times w + 0.03 \times w$
 $= 1.03 \times w$

Calculation : $762.20 = 1.03 w$
 $w = \frac{762.20}{1.03} = \740

1 Show what calculation you do to find these original amounts.

a) After an increase of 30%, an item costs \$91. Calculate the original cost (c) of the item.

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b) After a decrease of 10%, there are 396 students left. Calculate the original number of students (n).

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c) After a discount of 25%, a bowl costs \$26.55. Calculate the original price (p) of the bowl.

.....

d) After 15% GST is added, the TV costs \$4899. Calculate the price before GST (p).

.....



40 Factorising Quadratics 1

A Write with Brackets

Factorising is the opposite of expanding. When asked to factorise an expression you must write the expression with brackets.

Example : Factorise $10a^2 - 15a$

Working : Look for the greatest common factor in the terms.
 $10a^2 - 15a = 5a(2a - 3)$

1 Factorise.

- a) $3x + 12$
- b) $y^2 - 4y$
- c) $2a^2 + 6a$
- d) $4x^2 - 2x$
- e) $12y^2 + 8y$

An expression of the form $a^2 + 2a - 3$ is called a trinomial, it has 3 parts. Factorising a trinomial is the opposite of expanding double brackets.

Example : Factorise $a^2 + 2a - 3$

Working :

a^2	
	-3

 Place the expression in this box. Problem : the box has 4 parts, the expression has 3. But we know the two blank parts add to $2a$. Now find the numbers on the outside.

a^2 is made by $a \times a$ and -3 can be made by 1×-3 or by -1×3

	a	-3
a	a^2	$-3a$
1	$1a$	-3

 $-2a$ WRONG

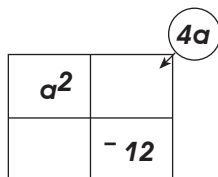
	a	3
a	a^2	$3a$
-1	$-1a$	-3

 $2a$ RIGHT

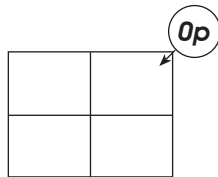
Answer : $a^2 + 2a - 3 = (a - 1)(a + 3)$
Note that $(a - 1)(a + 3)$ is the same as $(a + 3)(a - 1)$

2 Factorise these trinomials.

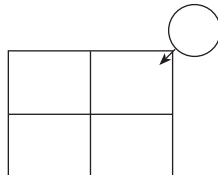
a) $a^2 + 4a - 12$
 $= (\dots)(\dots)$



b) $p^2 - 9$
 $= (\dots)(\dots)$



c) $2y^2 + 7y + 3$
 $= (\dots)(\dots)$



B Three Types of Quadratics

There are 3 types of quadratics to be factorised.

- (i) The common factor type which has two terms.
e.g. $4a^2 - 6a = 2a(2a - 3)$
- (ii) The trinomial which has three terms.
e.g. $2a^2 - a - 15 = (2a + 5)(a - 3)$
- (iii) The difference of two squares.
e.g. $4a^2 - 9 = (2a + 3)(2a - 3)$

1 Identify the type and factorise.

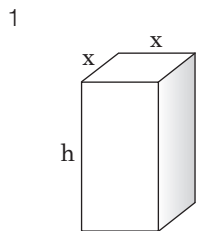
- a) $a^2 + 4a + 3$
- b) $x^2 - 25$
- c) $y^2 - 9y + 8$
- d) $p^2 + 6p$
- e) $q^2 - q - 2$
- f) $x^2 - 4x$
- g) $a^2 - 4$
- h) $y^2 - 4y - 32$
- i) $p^2 - 4p + 4$
- j) $t^2 - 20t + 36$
- k) $b^2 - 21b - 72$

2 Factorise.

- a) $3a^2 - 10a - 8$
- b) $9t^2 - 1$
- c) $20y^2 - 12y$
- d) $5x^2 - x - 6$
- e) $p^2 - p$
- f) $4 - a^2$
- g) $6p^2 - 7p - 3$
- h) $4y^2 + 8y + 3$
- i) $25a^2 - 16$
- j) $9x^2 + 14x + 16$
- k) $12x^2 - 31x - 30$

A A Numerical Approach

An optimal solution is a solution that maximises (or minimises) a quantity, while meeting certain conditions. We will use a numerical approach, which means constructing a table with possible solutions and selecting the optimal one.



1 An engineer is designing a small metal cuboid with a square cross section.

Optimal Solution : The engineer must find the dimensions of the cuboid with the largest possible volume.

Condition : The combined lengths of x and h must be 10 cm; $x + h = 10$

- Write a formula for the volume of the cuboid.
.....
- Suppose $x = 3$. What must h be? What will be the volume?
.....
.....
- Construct a table on a spreadsheet, choosing values of x and calculating corresponding values of h and V .

◇	A	B	C
1	x	h	Volume
2	1	9	9
3	2	8	32

Show the formulas in these cells :

A4	B4	C4

- If x and h had to be whole numbers, what would be the dimensions of the cuboid so that the volume is maximised?
.....
.....
- Adjust the table to find (to the nearest millimetre) the dimensions of the cuboid which will maximise the volume.

Complete :

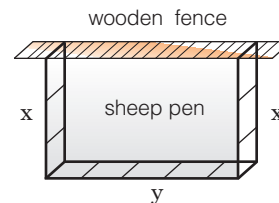
The optimal solution is a cuboid with dimensions :

$x =$ $h =$

This will give the maximum volume of cm^3 .

B Ba Ba Black Sheep

- A sheep farmer has 60 metres of electric fence which he uses to create a rectangular pen. One side of the enclosure is a wooden fence.



Problem :

Find the length (y) and width (x) which maximises the area of the pen, under the condition that $2x + y = 60$ metres.

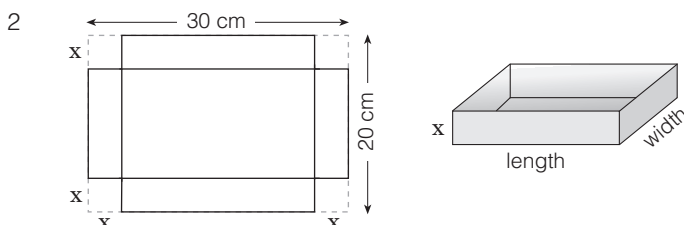
- Fill in values in the shaded cells for the chosen values of x .

◇	A	B	C
1	x	y	Area
2	1		
3	2		

Show the formulas in these cells :

A4	B4	C4

- Continue the spreadsheet. What is the optimal solution?
.....
- What is the largest possible area?



A rectangular piece of cardboard measuring 30 cm by 20 cm is used to make an open box. To do this, squares need to be cut from each corner. The sides of these squares become the height of the box.

We want to maximise the volume of the box.

- Set up a spreadsheet like this.

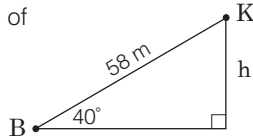
◇	A	B	C	D
1	x	length	width	volume
2				

- What is the optimal solution? What is the maximum volume?
.....
.....

A Sources of Error

When you write down a measurement you need to be aware of the precision you are implying.
 For instance, writing down a distance as 2.33 m means you measured to the nearest centimetre, 2.3 m means you measured to the nearest 10 cm.
 Revise : Measurement - pages 74 and 75.

- 1 A boy is flying a kite in the park. The string makes an angle of 40° with the ground, measured to the nearest 10 degrees. of the string is 58 m. Use the limits of accuracy to find the highest and lowest possible height of the kite above the ground (h).



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When we draw a diagram of a real-life situation, we are likely to make assumptions and to simplify the facts. This means we are introducing modelling errors.

- 2 Look at the diagram of the boy and his kite in the park.
 a) What assumption leads to the right-angle in the triangle?

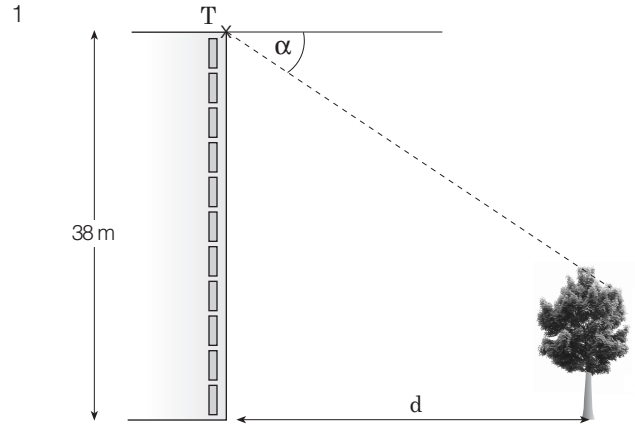
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- b) Find an other simplification. How would it affect h ?

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B An Assignment



Liam and Connor are lying on top of a building which is known to be 38 metres high. They want to find the height of the tree opposite the building.

- a) Liam measured the angle of depression α to be 38° , Connor reckons it is 31° . Give a possible reason for the variation.

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- b) This diagram shows the building and the tree as seen from above. Show with a line, where the boys should measure d .



- c) The boys found $d = 26.45$ m. What measurement should they write down? Say why.

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- d) Estimate the height of the tree.

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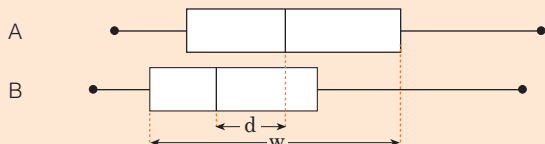
A Making the Call - 2nd Rule of Thumb

If there seems to be a large gap between two medians, but both medians are in the overlap of the boxes, then our first rule does not work. There is another rule of thumb :

Steps : i) find the difference between the two medians (=d).
ii) work out the overall visual spread, that is the total width of the boxes together (=w).

Rule : With a sample size of 30, you can make the call that the scores of population A are larger on average than those of population B, only if $d > \frac{1}{3}$ of w. With a sample size of 100, the call can be made if $d > \frac{1}{5}$ of w.

Example : Samples of size 30 were taken from two populations A and B. Can we claim that the scores of population A are on average higher than those of population B? Explain.



Working : We can see that d is about one quarter of w, this means that the difference between the medians is not large enough. The difference could just be sampling variation.

Conclusion : There is insufficient evidence that the scores of population A are larger on average than those of population B.

1a) Would the conclusion be different if the two samples A and B in the example, had a sample size of 100?

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b) What would be the reason that the rule changes for different sample sizes?

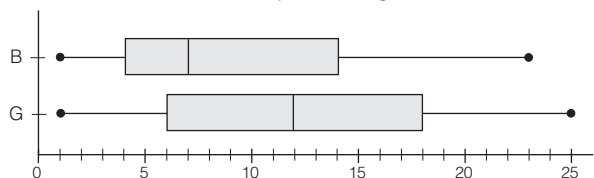
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2 Gemma and Benjamin did a survey to see whether Gemma sends longer text messages (more 'words') than Benjamin. They took random samples of 30 text messages on their phones. Study the two samples and write a conclusion.

Words per Message



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B Kiwi Fruit

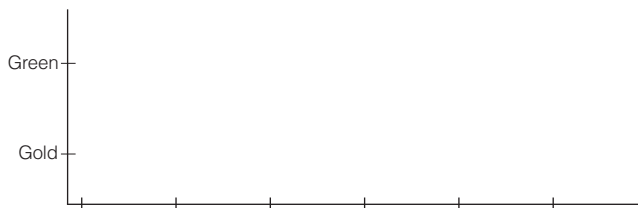
1 A grower has two varieties of kiwi fruit, green and gold. He took a random sample from each variety and measured the circumference of each fruit correct to the nearest mm.

green kiwifruit (mm)					gold kiwifruit (mm)				
152	144	161	128	148	144	136	128	149	139
150	143	165	136	142	126	138	146	135	148
137	149	160	154	146	139	145	138	122	125
130	128	143	153	163	130	124	118	126	131
144	127	138	140	115	129	144	139	124	133
133	138	118	122	136	141	153	145	141	135

a) Calculate the sample statistics that enable you to draw a box and whisker graph for each variety of kiwifruit.

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Circumference of Kiwifruit Varieties (mm)



b) Compare the distributions of circumferences of green and gold kiwifruit in the sample. Make a decision whether back in the orchard one type of fruit has a larger circumference.

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