

National Certificate of Educational Achievement - Level 3 Mathematics

Fast Track 5M

Mathematics Workbook

W. Geldof

Written for Year 13 students aiming to achieve
Level 3 NCEA Mathematics and Calculus standards

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1st Edition



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External Assessment - 6 credits

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Internal Assessment - 3 credits

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About this Book

A Choice of Standards

Sigma's Track workbooks are based on the New Zealand Mathematics and Statistics Curriculum and prepare students for NCEA assessments.

This mathematics workbook, *'Fast Track 5M'* is based on Level 8 Calculus and Mathematics strands of the NZ curriculum. Each chapter starts with some revision of Level 7, there are worked examples, a wide range of questions and problems, a practise test for each chapter and a full set of answers.

The book provides homework and revision work towards the following NCEA Level 3 *Mathematics* Achievement Standards :

- AS 3.1 - Geometry of Conic Sections..... 3 credits
- AS 3.2 - Linear Programming Methods..... 3 credits
- AS 3.3 - Trigonometric Methods.....4 credits
- AS 3.4 - Critical Path Analysis.....2 credits
- AS 3.5 - Algebra of Complex Numbers.....5 credits
- AS 3.6 - Differentiation Methods.....6 credits
- AS 3.7 - Integration Methods.....6 credits
- AS 3.15 - Systems of Simultaneous Equations3 credits

[black credits are internally assessed, orange credits indicate externally assessed standards.]

No Level 3 student is expected to do all eight standards, schools usually provide courses with a selection of five or six Achievement Standards (about 20-24 credits).

A **'Calculus Course'** typically contains AS 3.1, AS 3.3, AS 3.5, AS 3.6 and AS 3.7, with a total of 24 credits.

An **'Applied Maths Course'** could contain AS 3.1, AS 3.2, AS 3.3, AS 3.4, AS 3.5 and AS 3.15, with a total of 20 credits.

It is expected that students will use a range of technology throughout the year (graphing calculator, spreadsheet). This book contains some basic instructions for the *Casio fx9750 GII* calculator.

I wish you success with your studies this year.

Wiesje Geldof