5 Integers, Powers and Roots 2

A Brain Power

1 Caitlin wrote $2^3 = 6$. Show that this is not correct.

....

2 Calculate.

a) 3^4 ....

b) $(^-2)^2$ ....

c) $(^-1)^7$ ....

In the order of operations, powers are calculated after brackets but before any other operation.

3 Calculate.

a) $2^4 \div 4^2$ ....

b) $(3 - 5)^3$ ....

c) $^-6 + 3^2$ ....

d) $2^5 + (^-2 + 1)^3$ ....

e) $3^3 - 4^2 \div 2$ ....

f) $(3^3 - 4^2) \div 2$ ....

g) $2^3 (3^2 - 3) \div 2^4$ ....

4 Show that 6^3 must be the same as $2^3 \times 3^3$.

....

....

....

5 $9^2 = 81$. Show how you can use this fact to calculate 3^5.

....

....

....

6 You are asked to work out the missing number in $2^{\square} = 64$.
Complete this reasoning :
$8^2 = 64$ and $8 = 2 \times 2 \times 2$, therefore $2^{\square} = 64$.

7 Without a calculator, show how you can find the answer to
$2^6 \times 5^6$ ....

....

....

B Roots

The square root, written as $\sqrt{}$, is the reverse of squaring.
The cube root, written as $\sqrt[3]{}$, is the reverse of cubing.

Examples : Calculate. a) $\sqrt{81}$ b) $\sqrt[3]{125}$

Working: a) $\sqrt{81} = 9$, because $9^2 = 81$
b) $\sqrt[3]{125} = 5$, because $5^3 = 125$

1 Calculate.

a) $\sqrt{49}$ ....

b) $\sqrt{100}$ ....

c) $2\sqrt{9}$ ....

d) $\sqrt{(30 + 34)}$ ....

e) $\sqrt{9^2}$ ....

2 Calculate.

a) $\sqrt[3]{8}$ ....

b) $\sqrt[3]{1}$ ....

c) $\sqrt[3]{64}$ ....

d) $\sqrt[3]{^-8}$ ....

e) $\sqrt[3]{^-125}$ ....

3 Fill in the missing numbers.

a) $\sqrt{\ldots\ldots\ldots\ldots} = 20$ b) $\sqrt{\ldots\ldots\ldots\ldots} = 14$

c) $\sqrt[3]{\ldots\ldots\ldots\ldots} = 6$ d) $\sqrt[3]{\ldots\ldots\ldots\ldots} = 10$

4 Explain why $\sqrt{^-64}$ can't be found, but $\sqrt[3]{^-64}$ can.

....

....

....

....

5 Calculate.

a) $\sqrt{2^4}$ ....

b) $^-3\sqrt{25} \times 7\sqrt{4}$ ....

c) $\sqrt{5^2 + 3^2 \times 4^2}$ ....

9 Scientific Notation 2

A The Earth

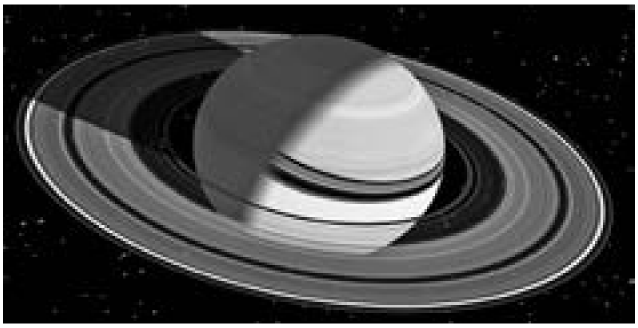
1 Write these numbers in scientific notation.

Earthly Facts (2 sf)	Scientific notation
equatorial circumference : 40,000 km	....
distance to sun : 0.000016 light years	....
2014 population : 7 200 000 000	....

2 Earth's mass is 5.98×10^{21} tonne. Saturn's mass is 5.68×10^{23} tonne. Which planet has a larger mass? About how many times larger?

....

....



3 Earth's average distance from the Sun is 1.5×10^{8} km. Mercury's average distance is 0.38 times Earth's distance. Calculate the distance of Mercury from the Sun. Give the answer in scientific notation.

....

....

....

4 The population of New Zealand is 4.2×10^{6} on a land area of 2.7×10^{5} km². How many people per km²?

....

....

....

5a) How many minutes in a year? ....

....

b) Round your answer to the nearest thousand.

....

c) Now write the answer in scientific notation.

....

B Atoms

To enter a number in scientific notation on your calculator you can use the EXP key.

Example : Calculate $(2.35 \times 10^{5}) \div (4.7 \times 10^{-7})$

Working : $\boxed{2.35}$ $\boxed{\text{EXP}}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{4.7}$ $\boxed{\text{EXP}}$ $\boxed{^{-}7}$

note : no brackets around the numbers

Answer on calculator : 5E+11 ; write : 5×10^{11}

On the recent models of Graphic Calculator, the EXP button looks like this $\boxed{\text{x}10^{x}}$.

1 Use the $\boxed{\text{EXP}}$ key on your calculator and write your answer in scientific notation.

a) $3.6 \times 10^{4} + 4.5 \times 10^{5}$ ....

b) $\dfrac{8.421 \times 10^{2}}{2.1 \times 10^{-1}}$ ....

c) $(6.5 \times 10^{-3})^{2}$ ....

2a) Calculate 15^{9}. Write the answer as a whole number.

....

b) Is the answer in a) exact? Explain your reasoning.

....

....

....

c) Write the answer to 15^{9} in scientific notation rounded to 4 sf.

....

3

Element	Symbol	Atomic Mass (g)
oxygen	O	2.66×10^{-23}
hydrogen	H	1.67×10^{-24}
calcium	Ca	6.65×10^{-23}
mercury	Hg	3.33×10^{-22}
helium	He	3.32×10^{-24}

a) Order these atoms from smallest to largest by mass :

H ,..

b) What is the mass of an H_2O molecule (water)?
(a water molecule = 2 hydrogen atoms plus 1 oxygen)

....

c) The atomic masses have been rounded. To what accuracy?

....

d) How many Ca atoms in 1 g of calcium? Round sensibly.

....

Working with Percentages 2

A A Percentage of an Amount

On a calculator you can find a percentage of an amount like this.
Change the percentage into a decimal (÷ 100) and
key in **of** as $\boxed{\times}$.

Example : Calculate 32% **of** $168.35.

Working : $\boxed{32}$ $\boxed{\div}$ $\boxed{100}$ $\boxed{\times}$ $\boxed{168.35}$ $\boxed{\text{EXE}}$

Answer : $53.87 (2 dp)

1 Calculate these. Round the answer to 2 dp.

a) 28% of $67.35 ....

b) 95% of $26.50 ....

2 Calculate these. Round the answer to the nearest whole.

a) 94% of 80 people ....

b) 72% of 120 houses ....

3 In Question 1 you were asked to round to 2 dp, in Q2 to the nearest whole. Explain why it is sensible to round that way.

....

....

....

....

....

4 Theo's income is $34 000. He pays income tax according to this rule : 10.5% tax on the first $14 000, 17.5% on any amount over $14 000. Calculate Theo's income tax.

....

....

....

....

....

5 According to Wikipedia (2014) the world population is 7.27×10^9 of which 0.525% live in Oceanian countries. How many people live in Oceanian countries? (round sensibly)

....

....

....

B Using Number Strategies

Use number strategies when you do percentage calculations.

Example : Work out a) 35% of $120
b) 95% of 80 kg

Working using number strategies :

a) 35% of $120 = 0.35 x 120
= 0.7 x 60 = $42

b) 95% of 80 = 100% of 80 - 5% of 80
Since 10% of 80 = 8, then 5% of 80 = 4
So 95% of 80 kg = 80 - 4 = 76 kg

1 Use mental strategies to calculate . . .

a) 30% of $120 ....

....

b) $66\frac{2}{3}$% of 24 kg ....

....

c) 5% of $175 ....

....

d) 98% of 325 kg ....

....

e) 37.5% of 4 L ....

....

2 Show that 18% of $25 must be the same as 25% of $18.

....

....

....

3 Show your strategies when you solve these problems.

a) The maximum score in a test was 75 marks.
Jeanine scored 88%. How many marks did she get?

....

....

....

b) A glasshouse has 425 glass panes, of which 48% were broken by a storm. How many panes were broken?

....

....

....

15 Goods and Service Tax

A Calculating GST

In New Zealand, the **Goods and Services Tax** (GST) is a value-added tax (VAT) that is added to the price of most goods and services. It is currently set at 15%. Businesses are required to add GST to the price of their goods and services when selling them, and they can also claim back the GST they paid on their business expenses.

GST calculations are very simple. To add GST to the cost of item, multiply by 1.15. To remove GST divide by 1.15. A price without GST or where GST removed is called the "GST exclusive" price. A price where GST has been added is called the "GST inclusive" price.

$$\text{GST exclusive price} \;\underset{\div\ 1.15}{\overset{\times\ 1.15}{\rightleftarrows}}\; \text{GST inclusive price}$$

Examples :

a)	Adding GST :	A wireless speaker costs \$200 before GST. Calculate the total price including GST. 200 x 1.15 = 230. The new price is \$230.
b)	Removing GST :	A laptop is purchased for \$1800, including GST. Calculate the GST exclusive price of the laptop. 1800 ÷ 1.15 = \$1565.22 (2 d.p.)

1 Find the answers to these questions.

a) Breakfast at a cafe costs \$80 exclusive of GST, how much is the total bill including GST?

………………………………………………………………………

b) A bicycle costs \$950 before GST.
How much is paid in total, including GST?

………………………………………………………………………

c) A television is priced at \$800 as a GST exclusive price.
Calculate the GST inclusive price.

………………………………………………………………………

d) A concert ticket costs \$175, and the price excludes GST.
Calculate the GST amount to be added to the ticket price.

………………………………………………………………………

e) The GST exclusive price of a hotel stay is \$325 per night.
How much is the GST inclusive price for a three night stay?

………………………………………………………………………

f) A camera costs \$450 before GST is added.
How much GST is added to the price?

………………………………………………………………………

g) A car rental agency charges \$80 per day before GST. If a car is rented for a week, what is the total cost including GST?

………………………………………………………………………

h) A restaurant bill comes to \$120 before GST is added to the total. Calculate the final bill including GST.

………………………………………………………………………

i) \$92 is paid for a pair of shoes, including GST. What is the price of the shoes excluding GST?

………………………………………………………………………

j) A gaming console is purchased for \$760, and this price includes GST. Calculate the price of the gaming console before GST was added.

………………………………………………………………………

k) The GST inclusive price for a book is \$28. What is the GST exclusive price of the book?

………………………………………………………………………

l) A bicycle costs \$1050, including GST.
Find the price of the bicycle before GST.

………………………………………………………………………

m) An invoice is received from an electrician for \$460, which includes GST. Find the amount of GST included in the invoice.

………………………………………………………………………

n) A smartphone is bought in a store for \$600, excluding GST. In addition, a charger was purchased for \$20, excluding GST. Calculate the total cost of both items, including GST.

………………………………………………………………………

o) A business spends \$500 on office supplies, and this amount includes GST. How much GST can the business claim as a refund if they are registered for GST?

………………………………………………………………………

Using a Ratio Table 1

A Being Directly Proportional

The amounts in a baking recipe change in the same proportion if we want to make more (or less) than the standard amount.

A problem in which one set of numbers is multiplied by a **constant**, giving a second set of numbers, is called a **directly proportional** problem.

A **ratio table** consists of rows of numbers and each row is directly proportional to the previous.

Example : Here is a table with 2 rows and 3 columns, we call it a 2x3 table.

a) Show that the 2nd row is directly proportional to the 1st row.

b) Blake says, *"Also each column is directly proportional to the previous."* Is that true?

2	4	10
6	12	30

Working :

a) The second row is 3 times the first row.

b) Yes (see arrows).

x2 x$2\frac{1}{2}$

x3

2	4	10
6	12	30

x5

1 Work out the scale factors in these ratio tables.

a) x ....

x ....

4	20
8	40

b) x .... x ....

x ....

1	3	$1\frac{1}{2}$
5	15	$7\frac{1}{2}$

2 Use a scale factor to complete these ratio tables.

a) x ....

6	18
15	

b) x ....

x ....

3	5	
	35	70

B Reading the Table

1 Here is a simple 2x2 ratio table. Use the numbers in the table to write six true number sentences. A start has been made.

2	3
8	12

i) $\frac{2}{3} = \frac{8}{12}$ ii) 8 ÷ 2 = 12 ÷ 3

iii) ………………………… iv) …………………………

v) ………………………… vi) …………………………

2 Select numbers from this ratio table to complete the sentences.

1	6	4	7.5
2.5	15	10	18.75

a) 15 ÷ 6 = 10 ÷ …………… b) 1 x 10 = ……… x ………

c) $\frac{10}{15}$ = $\frac{\dots\dots}{\dots\dots}$ d) 6 x 18.75 = ……… x ………

e) 7.5 ÷ 4 = ……… ÷ ……… f) 2.5 : 15 = ……… : ………

In column Ⓐ we used scale factors to find missing numbers in a table. Working out these factors is not always easy. Now you will see other simple strategies to work out missing numbers.

3

5	4	7
a	2.5	b

This is a 2x3 ratio table. We will calculate the values of a and b. Complete the working.

a) In this table 4 x a = 5 x 2.5 or 4a = 12.5

Calculate a ……………………………………………

b) In this table $\frac{b}{7} = \frac{2.5}{4}$ or $\frac{b}{7}$ = 0.625

Calculate b ……………………………………………

C Missing Numbers

1 Calculate the missing numbers in these ratio tables.

a)

21	15		27
14		6	

……………………………………………

……………………………………………

……………………………………………

……………………………………………

……………………………………………

……………………………………………

b)

35		63
	16	18
	20	

……………………………………………

……………………………………………

……………………………………………

……………………………………………

……………………………………………

2 Make up a ratio table for this problem, then calculate w.

5 : w = 12 : 8

w = …………

25 Financial Literacy 2

A Earning Interest

Suppose you go to the bank to invest an amount of money, $\$P$. The bank will tell you the interest rate, $R\%$, per annum. If the amount invested does not change (i.e. no extra money is added to the investment) then the simple interest at the end of T years is found with this formula :

$$I = P \times \frac{R}{100} \times T \quad \text{or} \quad I = \frac{PRT}{100}$$

Example : When baby Chloe was born, her dad invested $1000 at an interest rate of 4.5% per annum. No more money was added. How much simple interest does $1000 earn in 3 years at this interest rate?

Working : $I = (1000 \times 4.5 \times 3) \div 100 = 135$; answer $135

1 How much simple interest is received if an amount of $5000 is invested at 4.8% interest per annum for 4 years?

....

....

2 What scenario earns more simple interest, A or B?

A : $2400 invested for 5 years at 6.0% interest per annum.

B : $3000 invested for 6 years at 4.0% interest per annum.

....

....

....

3 Amy invested $4000 at 5% p.a. After 1 year Amy deposited the interest she had earned back into her investment account.

a) How much money was in Amy's investment account after year 1?

....

....

b) Another year goes by, Amy's new investment earns 5% interest. How much interest does she get this 2nd year?

....

....

c) If all interest stays in the investment account, what would be the total in Amy's account at the end of the 3rd year?

....

....

....

....

....

B Compound Interest

Banks advertise their interest rates per annum, but they usually calculate the interest monthly.
If the interest stays in the savings account then the amount in the account is growing and so is the amount of interest. This is called **compound interest**.

Example : A sum of $800 is invested in a savings account with an interest rate of 5% p.a. compounded monthly.

a) Calculate the interest received after one month.
b) Calculate the amount in the bank after one month.
c) Calculate the amount in the bank at the end of the year with compound interest.

Working : 5% per year $= \frac{5}{12}\%$ per month (or $0.0041\overline{6}$)

a) interest $= 0.0041\overline{6} \times \$800 = \$3.33$
b) amount $= 1.0041\overline{6} \times \$800 = \$803.33$
c) amount in the bank after

1 month $1.0041\overline{6} \times \$800$
2 months $1.0041\overline{6} \times 1.0041\overline{6} \times \$800$
12 months $(1.0041\overline{6})^{12} \times \$800 = \$840.93$

1 $10 000 is invested with compound interest at 6% per annum, calculated monthly. Calculate . . .

a) the amount in the bank after 1 month.

....

....

b) the amount in the bank after 18 months.

....

....

2 Calculate the amount in the bank if $25 000 is invested with compound interest at 4% p.a. (calculated monthly) for 3 yrs.

....

....

....

....

3 Bernadette borrowed $4000 from her uncle at 3% interest per annum, compounded monthly. Now, half a year later, she is earning enough money to start repaying the debt. How much does she owe now?

....

....

....

....

33 Rates 2

A Speed

Examples :

a) Jason drove 7 km in 4 minutes. Find Jason's speed in km/h.

b) Rebekah drove 115 km at an average speed of 90 km/h. How long did the trip take?

Working :

a) First divide by 4 to get the distance covered in 1 minute, then multiply by 60 to get distance covered in 1 hour.

7 km in 4 min $= \frac{7}{4}$ km in 1 min $= 60 \times \frac{7}{4}$ km in 1 hour.

Answer : 105 km/h

b) Every hour Rebekah drives 90 km, so 115 km is covered in $\frac{115}{90}$ hours.

$\frac{115}{90}$ hours $= 1\frac{5}{18}$ hours = 1 hour 17 min

1 An aircraft flies at a speed of 405 km/h.

a) How far does it fly in 25 minutes?

....

....

....

b) How long does it take to fly 108 km?

....

....

....

2 A cyclist covers a distance of 12 km in 25 minutes.

a) What is her average speed?

....

....

b) At this speed, what distance would she cover in 35 minutes?

....

....

3 Jacob drove 125 km at an average speed of 80 km/h and 75 km at 90 km/h. How long did the trip take? Give your answer in hours and minutes.

....

....

....

....

....

B Changing Units

Example :

An orca can reach a speed of 15 metres per second. Change that speed into km/h.

Working :	15	metres per second
	= 60 x 15	metres per minute
	= 60 x 60 x 15	metres per hour

Answer : 54 000 m/h = 54 km/h

1 Car A goes at 80 km/h, car B goes at 20 m/s.

Which car goes faster? ....

....

....

2 It takes the driver of a car one second to react to whatever is happening on the road ahead.

a) If his speed is 50 km/h, how many metres would the driver have covered in one second? Complete this working :

50 km/h = 50 000 metres in 60 minutes

= .... m/min = ... m/s

b) What if he was going 65 km/h? .... m/s

3 A cheetah can reach a speed of 95 km/h but can only keep that up for 22 sec. How far does the cheetah run in 22 sec at top speed?

....

....

....

4 Road works are in progress on a stretch of road 800 m long. The speed limit is 30 km/h. A road-worker found that many cars take 50 seconds to travel down this stretch of road. How much over the speed limit do these cars go?

....

....

....

....

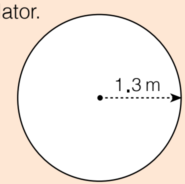
....

....

Area of Basic Shapes 4

A Formula

The formula for the area of a circle is $A\bigcirc = \pi \times r^2$
where A is the area, r the length of the radius and
$\pi = 3.14159265.....$ as found on your calculator.



Example :

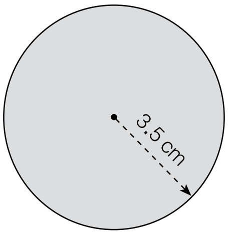
The area of this circle is

$A = \pi \times 1.3^2 = 5.30929158.....$
$= 5.3\text{ cm}^2$ (2 sf)

Notes :
- 1.3^2 may be keyed in as 1.3 x 1.3 or as 1.3 $\boxed{x^2}$
- round your answer
- the unit for area is cm^2, m^2, etc

1 Calculate the area of these circles. Round to 2 significant figures and remember to write the unit.

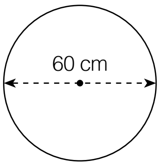
a)



……………………………

A = ……………………..

b)

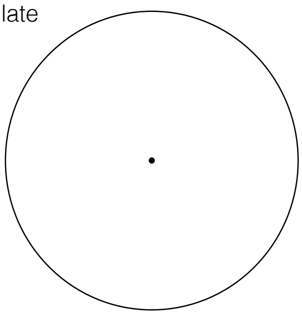


r = ……………………..

……………………………

A = ……………………..

2 Take measurements and calculate the area of this circle to the nearest whole cm^2.



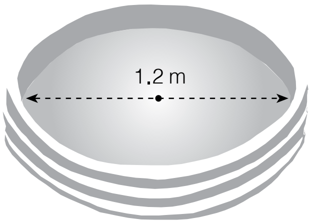
……………………………

……………………………

……………………………

A = ……………………………

3



The diameter of this paddling pool is 1.2 m.

Calculate the area of its floor.

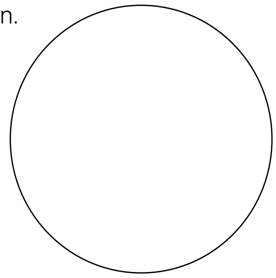
Give your answer in m^2 rounded to 2 sf.

…………………………………………………………

…………………………………………………………

B Semi-Circles

1 The centre of this circle is not shown.



a) Estimate the diameter and then the radius.

……………………………

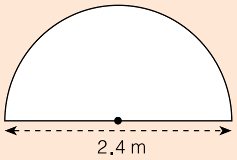
……………………………

b) Estimate the area of the circle.

…………………………………………………………

When a circle is cut in half, the cutting line is always a diameter of the circle. The area of a semicircle is half of the area of the circle. $A = \frac{1}{2} \times \pi \times r^2$

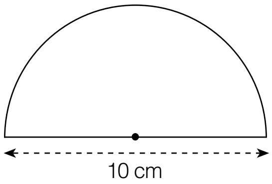
Example :



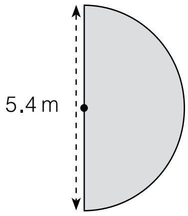
diameter = 2.4 m
radius = 1.2 m
Area = $0.5 \times \pi \times 1.2^2$
= 2.3 m^2 (2 sf)

2 Calculate the areas of these semicircles. Round to 2 sf.

a) diameter = ……………………
radius = ……………………
Area = ……………………
= ……………………

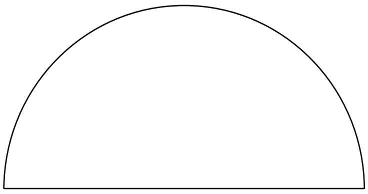


b) diameter = ……………………
radius = ……………………
Area = ……………………
= ……………………



3a) Measure the diameter.

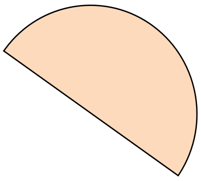
……………………………



b) Calculate the area of this semicircle.

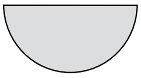
…………………………………………………………

4 Calculate the area of each semicircle.



a) ……………………………………

b)



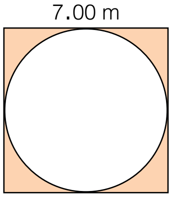
…………………………………………………………

45 Area of Composite Shapes 3

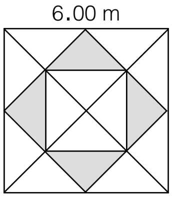
A Inside the Square

1 Show how you calculate the shaded areas.

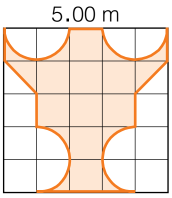
a)



b)



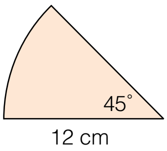
c)



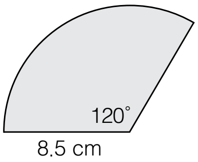
B Sectors of Circles

1 Calculate the perimeter of these sectors.

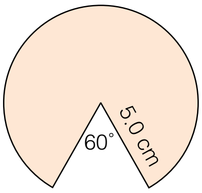
a)



b)

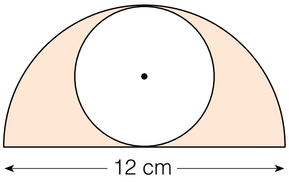


2 Calculate the area of this sector.

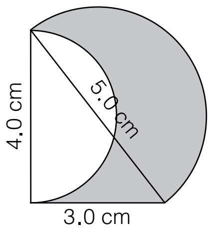


C Holes and Bulges

1 Calculate the coloured area of this shape.



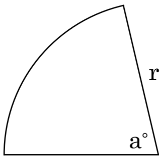
2 Calculate the area of the coloured part.



D Make Up a Formula

1 Write a formula for the area of these shapes, using the letters shown in the shape.

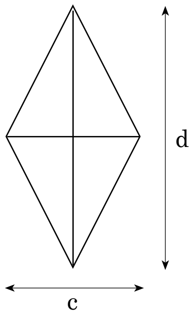
a)



The area of a sector with angle a° and radius r.

formula : A = ……………………………………………………………

b)



The area of a rhombus with diagonals of length c and d.

formula : A = ……………………………………………………………

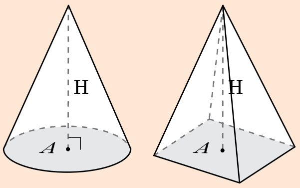
51 Volume of Basic Solids 3

A Changing Cross-sections

Cones and pyramids are solids with an ever changing cross-section, they end in a point called the apex. Their volumes are found with the formula

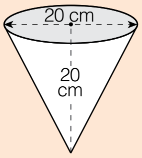
$V = \frac{1}{3} \times A \times H$,

where A is the area of the base and H the perpendicular height from the base to the apex.



Example :

How many litres of water could this cone hold?



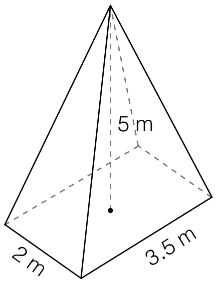
Working : $r = 10$; $A = \pi \times 10^2$;
$H = 20$.
$V = \frac{1}{3} \times \pi \times 10^2 \times 20$

key in : $\boxed{1}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{\times}$ $\boxed{\pi}$ $\boxed{\times}$ $\boxed{10}$ $\boxed{x^2}$ $\boxed{\times}$ $\boxed{20}$ $\boxed{\text{EXE}}$

Answer : $V = 2094 \text{ cm}^3 = 2.1$ litres (2 sf)

1 Calculate the volumes of these solids.

a)



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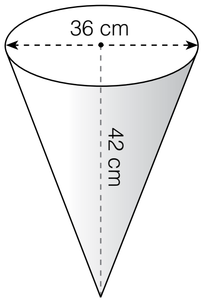
b)



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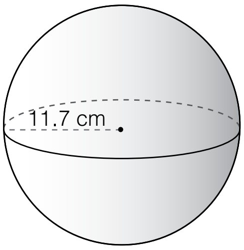
The volume of a sphere is found with the formula $V = \frac{4}{3}\pi r^3$.

2



A basketball has a radius of 11.7 cm. Calculate the volume of the basketball.

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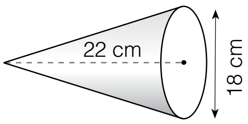
B More Practice

1 Calculate the volume of these solids.

a)



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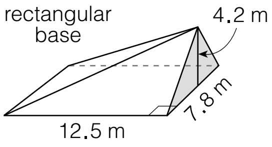
b)



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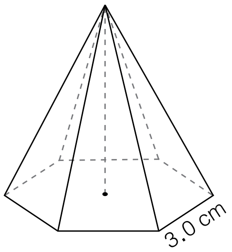
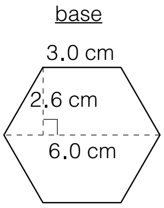
2





This pyramid is 7.5 cm high, it has a regular hexagon as its base. Calculate the volume of the pyramid.

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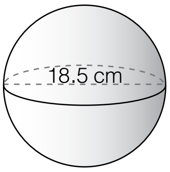
....

3 Calculate the volume of these solids.

a)



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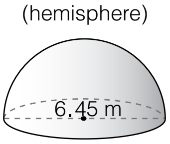
b)



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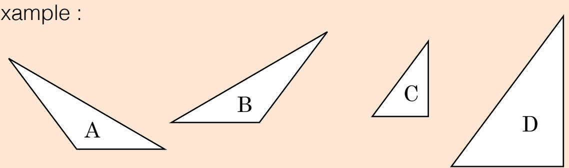
55 Similar Shapes 1

A Congruent or Similar?

Congruent figures have the same shape and the same size. By rotating them or flipping them over you can fit congruent shapes exactly on top of each other.

Similar figures have the same shape but not the same size. The sides of the smaller figure are multiplied by a **factor** (k) to get the sides of the larger shape.

Example :

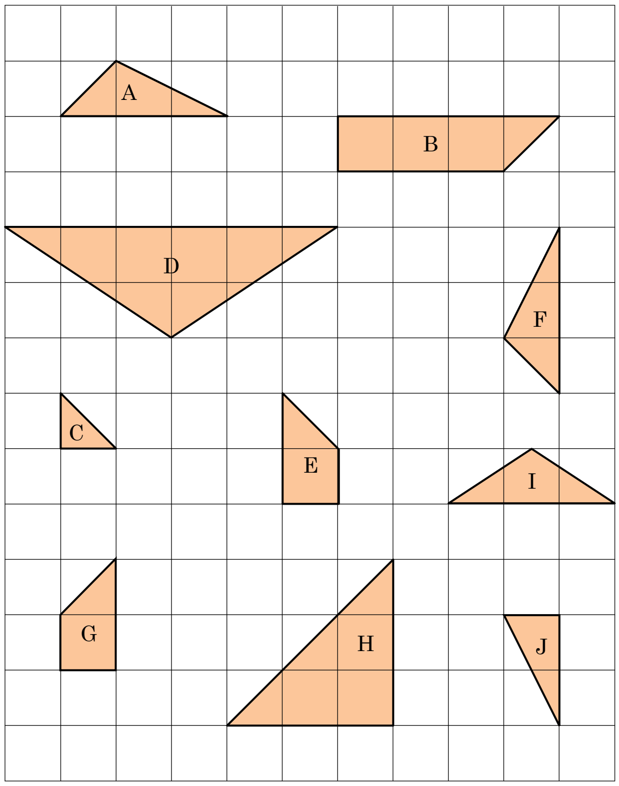


Figures A and B are congruent.
Figures C and D are similar. The scale factor for the sides is 2 (k = 2).

1a) Name pairs of congruent shapes in the diagram below.

A and …………; ………………………………………………………

…………………………………………………………………………



b) Name pairs of similar shapes.
For each pair, give the scale factor k.

…….. and …….., k = ………; …..………………………….;

…………………………………………………………………………

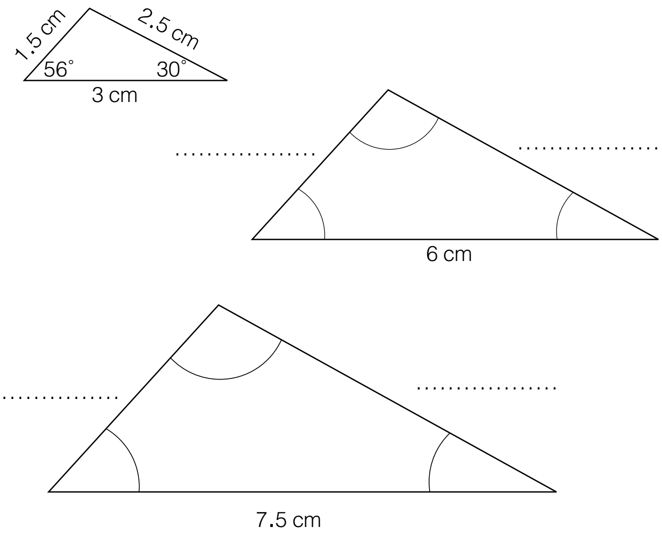
…………………………………………………………………………

…………………………………………………………………………

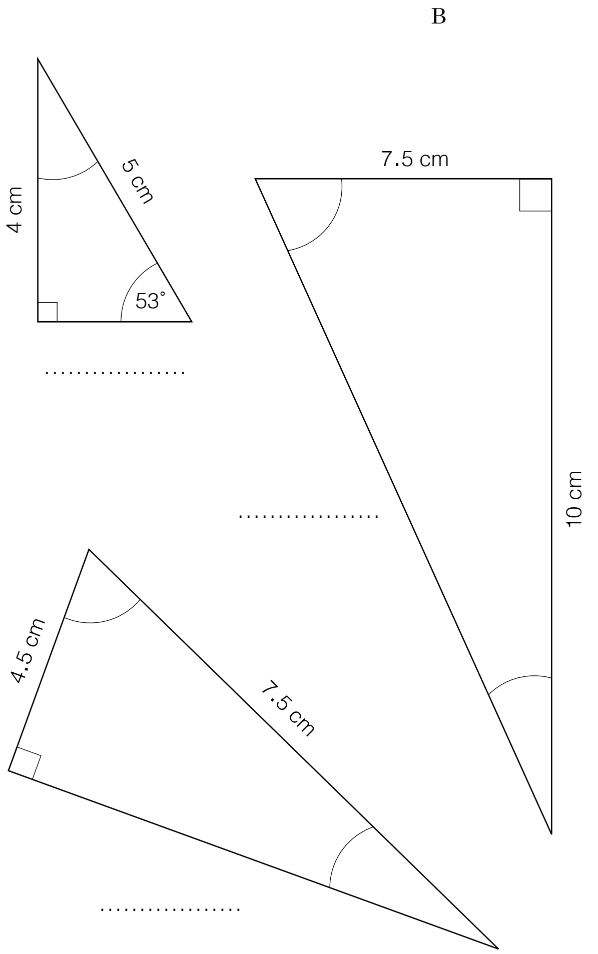
B Angles and Sides

Remember : Angles inside a triangle add to 180˚

1a) These three triangles are similar (they are not drawn to scale). Work out all sides and all angles of the triangles.



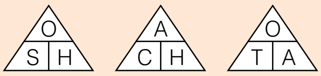
b)



Basics of Trigonometry 1

A Four Steps to Calculate a Side

If in a right-angled triangle you know the size of one more angle and the length of one side, then you can use the ratio triangles to calculate any of the other two sides.

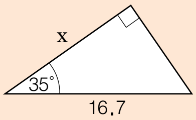


The calculation has 4 steps :

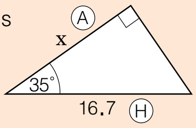
1) In the right-angled triangle label two of the sides with **H** (hypotenuse) or **O** (opposite) or **A** (adjacent). Only label the side you know and the side you want to know.
2) Choose the relevant ratio triangle : SOH, CAH, or TOA.
3) Substitute known values into the ratio triangle.
4) Calculate the length of the side using your calculator and round sensibly.

Example : Calculate x.

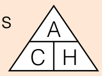
Working :



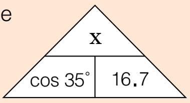
1) Labels



2) With labels A and H the choice is



3) Substitute values.

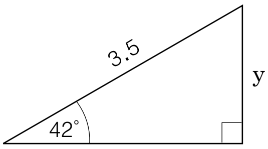


4) Calculate and round.

$x = \cos 35^\circ \times 16.7$

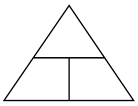
$= 13.7$ (3 sf)

1 We will use the 4 step method to calculate the length of side y.

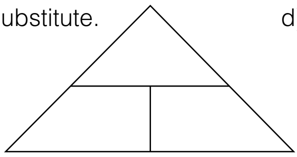


a) Label sides 'y' and '3.5'.

b) Choose SOH, CAH or TOA.



c) Substitute.



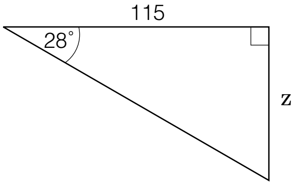
d) Calculate and round y.

y = ...

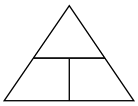
....

2 Calculate the length of side z in four steps.

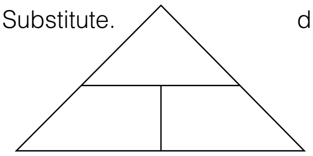
a) Label sides 'z' and '115'.



b) Choose SOH, CAH or TOA.



c) Substitute.



d) Calculate and round z.

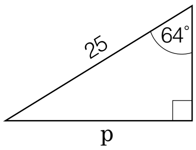
z = ...

....

B On Your Own

1 Calculate the labelled sides, round sensibly.

a)



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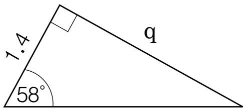
b)



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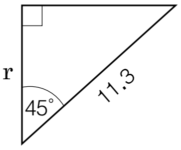
c)



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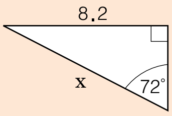
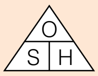
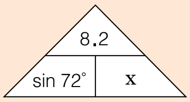
Example : Calculate side x.

Working : x has label H
8.2 has label O



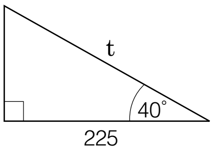




$$x = \frac{8.2}{\sin 72^\circ} = 8.6 \text{ (2 sf)}$$

2 Calculate the labelled sides, round sensibly.

a)



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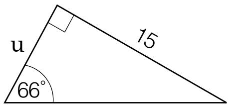
b)



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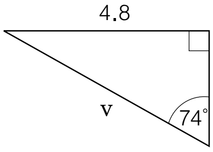
c)



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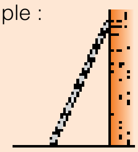
71 Applications of Trigonometry 1

A Labels and Units

In a word problem the side to be calculated is not usually marked with x, you have to do that yourself.

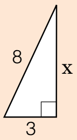
The unit of measurement is important. Check that all given measurements have the same unit, the answer will get that same unit. While doing the calculations however, you ignore the unit and work with numbers only.

Example :



A house painter has a ladder which extends to 8 m.
The foot of the ladder is placed 3m from the building.
How high up the building does the ladder reach?

Working :



$$x^2 + 3^2 = 8^2$$
$$x^2 = 8^2 - 3^2$$
$$x = \sqrt{8^2 - 3^2}$$
$$x = 7.4 \ \text{(1dp)}$$

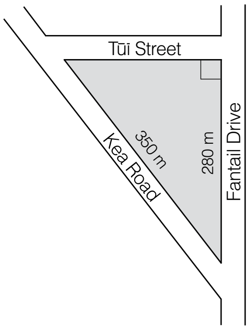
ignore the unit

write the unit

Answer : The ladder reaches 7.4 m (2 sf) up the building.

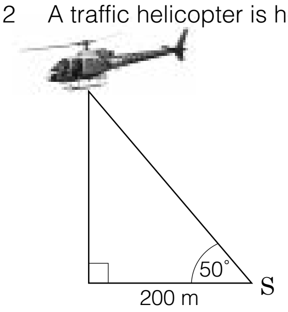
1 This map shows a park surrounded by three roads.

Calculate the length of Tūī street.



Answer : ....

2 A traffic helicopter is hovering above a roundabout.



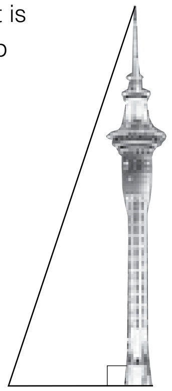
Sasha is 200 m away from the roundabout. When looking up at the helicopter, the angle above the horizontal is 50°.

Calculate the height of the helicopter above the ground.

Answer : ....

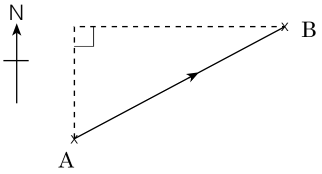
B Place the Measurements

1 The Auckland Skytower is 328 m high. What is the angle between the horizontal and the top of the Skytower at a point 90 m from the entrance?



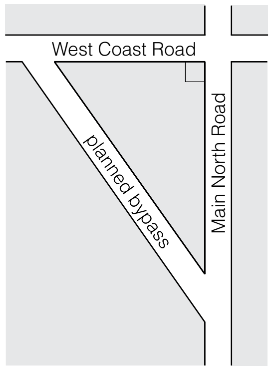
Answer : ....

2 Airport B is 150 km North and 220 km East of airport A.
A plane takes off at A and lands at B.
What distance did it fly?



Answer : ....

3



The council is planning a bypass to relieve the busy intersection between West Coast Road and Main North Road. The bypass will make an angle of 35° with Main North Road and it will join West Coast Rd 2.5 km from the intersection.

Calculate the length of the bypass.

Answer : ....

Plotting Lines 1

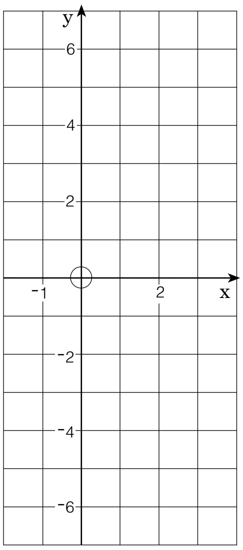
A Plotting a Graph

With an equation we can make a table, and with a table we can make a graph. If the x-value can be any (decimal) number, then the graph is a continuous line.

1 Make a table for each rule then plot the points and connect them. There will be 2 graphs on each grid.

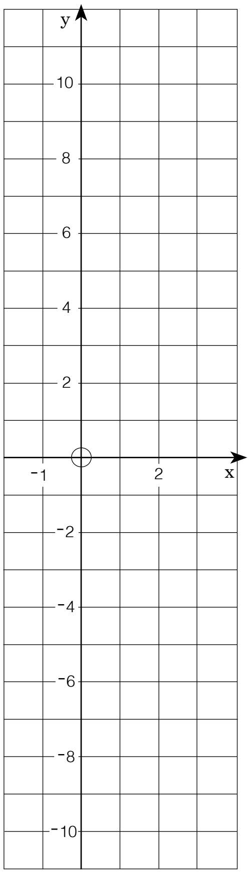
a) $y = {}^{-}x + 2$

x		y
$^{-}1$		
0		
1		
2		



b) $y = \frac{x - 3}{2}$

x		y
$^{-}1$		
0		
1		
2		



c) $y = 5 - 3x$

x		y

d) $y = 4(x - 1)$

x		y

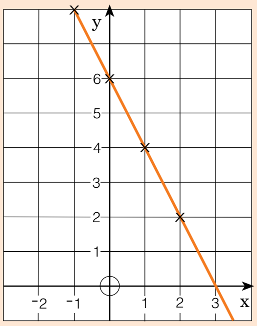
B Rules in the Form ax + by = c

Example : a) Make a table for the rule $2x + y = 6$
b) Plot the graph.

Working : a) Substitute x, into the rule, calculate y.

x	$2x + y = 6$	y	$(x,\ y)$
$^{-}1$	$^{-}2 + y = 6$	8	$(^{-}1,\ 8)$
0	$0 + y = 6$	6	$(0,\ 6)$
1	$2 + y = 6$	4	$(1,\ 4)$
2	$4 + y = 6$	2	$(2,\ 2)$

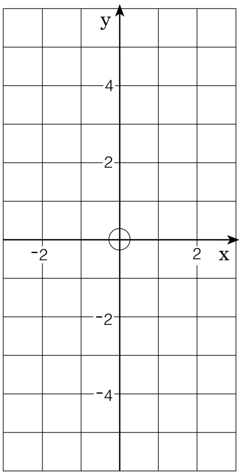
b) Plot the points and draw the line.



1 Make a table for each rule then draw the graph. There will be 2 graphs on one grid.

a) $3x - y = 1$

x		y
$^{-}1$		
0		
1		
2		

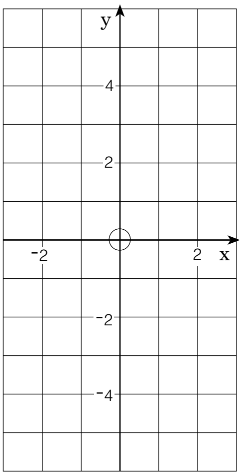


b) $x + 2y = 8$

x		y
$^{-}1$		
0		
1		
2		

c) $2x + \frac{1}{2}y = 1$

x		y
$^{-}1$		
0		
1		
2		



d) $3x - 2y = 0$

x		y
$^{-}1$		
0		
1		
2		

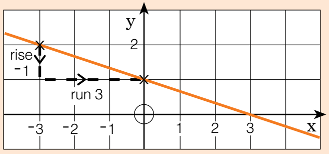
87 Features of Line Graphs 1

A How Steep?

The **gradient** of a line is a fraction that indicates its **steepness**.
The gradient is calculated as follows :

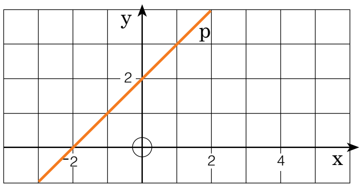
step 1 Fix two points on the line.
step 2 Move from one point to the other over the grid.
step 3 Count the squares to find the rise and the run.
step 4 gradient = $\frac{\text{rise}}{\text{run}}$ (simplify the fraction)

Example :



gradient = $\frac{-1}{3}$ (or $-\frac{1}{3}$)

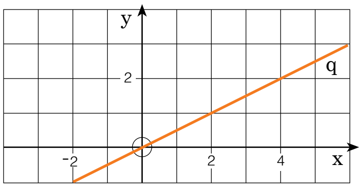
1 Study the graphs and work out their gradients.



a) gradient line p

....

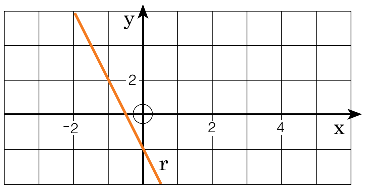
....



b) gradient line q

....

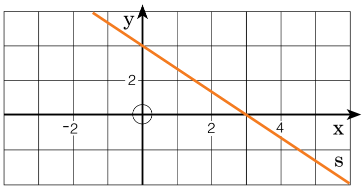
....



c) gradient line r

....

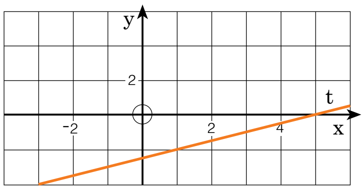
....



d) gradient line s

....

....



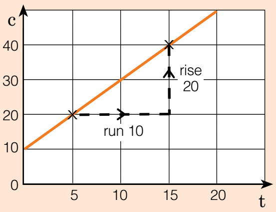
e) gradient line t

....

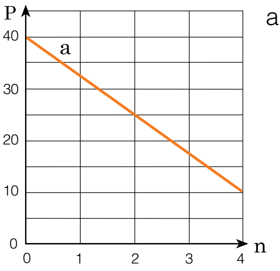
....

B Real Life

In real life graphs the scales on the axes are often different.
In this example
gradient = $\frac{20}{10}$ = 2.



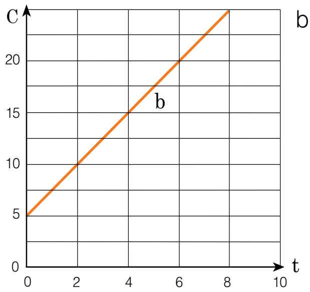
1 Carefully work out the gradient of these lines.



a) gradient line a

....

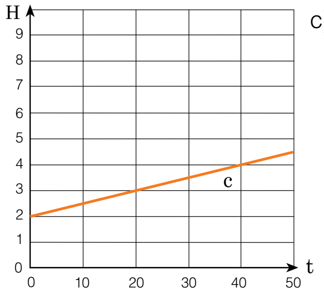
....



b) gradient line b

....

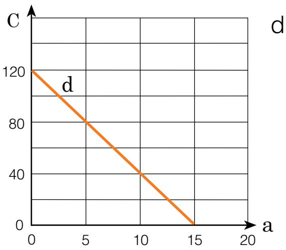
....



c) gradient line c

....

....



d) gradient line d

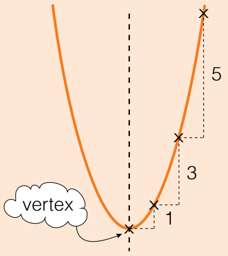
....

....

Plotting Parabolas 2

A Looking for a Pattern

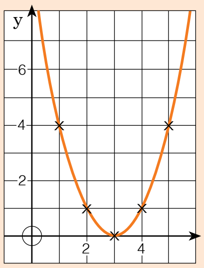
The parabolas plotted so far have the same shape.
When plotting crosses you may have noticed a *forward difference pattern* : starting at the vertex and moving from one cross to the next the pattern is **'out 1, up 1'**, **'out 1, up 3'**, **'out 1, up 5'**, etc.
This pattern is the same on either side of the vertex.



So if you know the coordinates of the vertex, you can graph a parabola without making a table.

Example : Plot the graph of
$y = (x - 3)^2$.

Working :
The vertex is at (3, 0). Plot the vertex and go **'out 1, up 1'** etc. to get the other crosses.
Join with a curve.



1 For each of the parabolas, plot the vertex.
Work out where the crosses go and draw the graph.

a) $y = x^2 + 4$

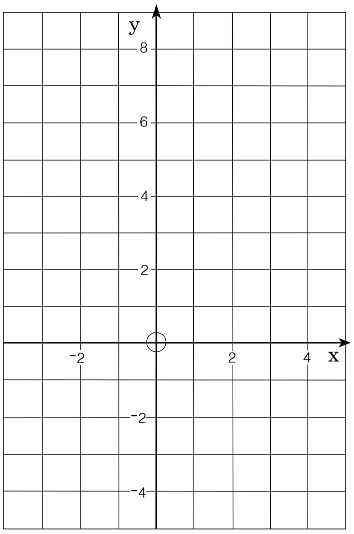
vertex (………, ………)

b) $y = x^2 - 3$

vertex (………, ………)

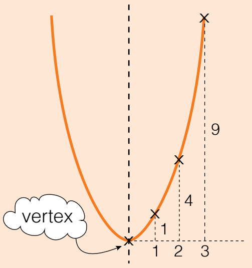
c) $y = (x + 1)^2$

vertex (………, ………)



Another pattern you may have noticed is the pattern of square numbers. For this pattern we count how far each cross is from the vertex.

The first cross is **'out 1, up 1'**, from the vertex, the second is **'out 2, up 4'**, the third is **'out 3, up 9'**, etc.



2 Check that the graphs you plotted in question 1 above show the pattern of square numbers.

B The Graphic Calculator

We can use the graphic calculator to draw graphs for us and to find features. The following steps work for a *Casio fx 9750* or *9860 GIII* graphic calculator.

Example :

a) Where is the vertex of the parabola $y = (x + 4)^2 - 1$?

b) Where does this parabola cut the axes?

Working : Select **GRAPH** from the main menu.

Note : Make sure TYPE is $y =$
with **DEL** delete any unwanted equations.
Now type in the equation

$\boxed{(}\ \boxed{X,\theta,T}\ \boxed{+}\ \boxed{4}\ \boxed{)}\ \boxed{x^2}\ \boxed{-}\ \boxed{1}\ \boxed{\text{EXE}}$

Select **DRAW** (F6)

Note : It may be necessary to adjust your viewing window.
Press (F3) V-Window and let x go from ⁻8 to 8, scale 1, and let y go from ⁻5 to 5, scale 1. Press (EXIT) .

Once your graph looks good, press (F5) G-Solv and select the features you wish to find.

ROOT gives the x-intercepts
MIN gives the vertex (minimum value)
Y-ICPT gives the y-intercept

Check that you get these answers :

a) vertex is at (⁻4, ⁻1)

b) x-intercepts at (⁻5, 0) [press **REPLAY** ▶] and (⁻3, 0)
y-intercept at (0, 15)

1 Find vertex and intercepts of each graph.

a) $y = (x - 2)^2 - 4$

…………………………………………………………………………

…………………………………………………………………………

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b) $y = (x + 1)^2 + 2$

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…………………………………………………………………………

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c) $y = (x + 5)^2 - 4$

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

103 Factorised Equations 1

A Smart Plotting

The equation $y = (x - p)(x - q)$ is also a quadratic equation. Its graph is a parabola.

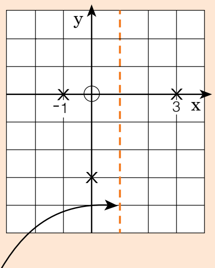
Example : We will plot the parabola $y = (x + 1)(x - 3)$ by working out its special features. Each time we find some coordinates we put them on the grid.

a) Work out the y-intercept.
b) Work out the x-intercepts.
c) Work out the coordinates of the vertex.
d) Sketch the graph.

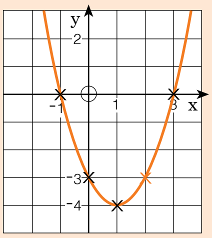
Working : $y = (x + 1)(x - 3)$

a) For the y-intercept, make $x = 0$.
Then $y = (0 + 1)(0 - 3) = {}^{-}3$
Plot point $(0, {}^{-}3)$

b) For the x-intercepts, make $y = 0$.
Solve : $(x + 1)(x - 3) = 0$
$x = {}^{-}1$ or $x = 3$
Plot points $({}^{-}1, 0)$ and $(3, 0)$



Now we can draw the line of symmetry for the parabola. The line must go halfway between the two x-intercepts. The vertex must be on this line.



c) The x-coordinate of the vertex is at $x = 1$, then
$y = (1 + 1)(1 - 3) = {}^{-}4$
Plot the vertex at $(1, {}^{-}4)$

d) Use symmetry to plot another point.

1 Take these steps to plot the parabola $y = (x + 2)(x - 4)$.

a) Find the y-intercept :

$x = 0$

$y =$ ……………………………

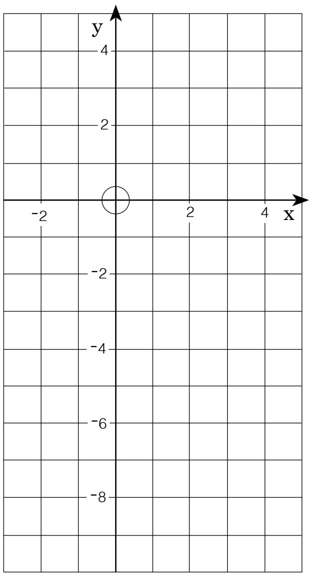
b) Find the x-intercepts :

$y = 0$

$(x + 2)(x - 4) = 0$

$x =$ …………… or ……………

c) Plot the intercepts and draw the line of symmetry.



d) Calculate the coordinates of the vertex.

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……………………………………………………………………

e) Draw the parabola.

B Intercepts and Vertex

For each parabola work out the intercepts with the axes, find the vertex, then sketch the graph.

1 $y = (x + 2)(x + 4)$

……………………………………

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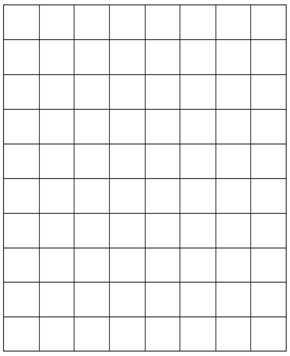

2 $y = (x - 2)(x + 3)$

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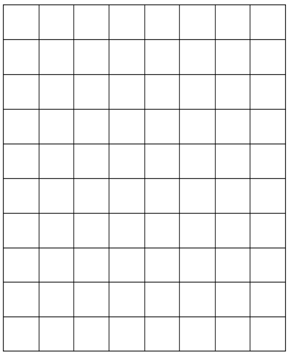

3 Calculate the coordinates of the vertex of . . .

a) the parabola $y = (x + 3)(x - 5)$.

……………………………………………………………………

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b) the parabola $y = x (x + 3)$.

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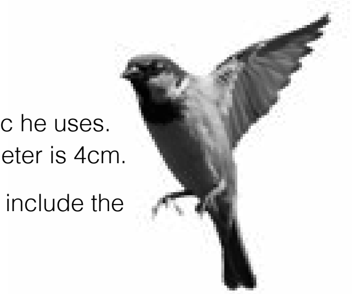
C Max’s Bird Feeder - Task C

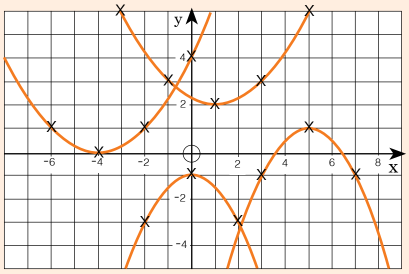


Max wants to investigate different sizes of cylinder to see what effect this has on the amount of plastic he uses. His diameter plus height must remain at 28cm and be an even number. The minimum height or diameter is 4cm.

Using a graph and/or table - find the dimensions for the cylinder which uses the most plastic. Do not include the plastic for the hemisphere in your calculations.

Page 101 - Wide and Narrow Parabolas 1 - cont.

B2 a), b), c), d).



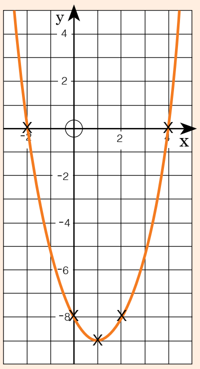
Page 102 - Wide and Narrow Parabolas 2

A1 a) vertex : $(^-2, ^-3)$; pattern : out 1, up 2, out 2, up 8
b) vertex : (0, 1); pattern : out 1, down $\frac{1}{4}$, out 2, down 1, out 3, down $2\frac{1}{4}$, out 4, down 4
c) vertex : $(^-1, 1)$;
pattern : out 1, up $\frac{1}{2}$, out 2, up 2, out 3, up $4\frac{1}{2}$

B1 a) vertex : $(^-2, ^-4)$;
pattern : out 1, up $\frac{1}{2}$, out 2, up 2
$y = \frac{1}{2}(x + 2)^2 - 4$
b) vertex : (1, 7);
pattern : out 1, down 2, out 2, down 8
$y = {^-2}(x - 1)^2 + 7$
c) vertex : $(3, ^-7)$;
pattern : out 1, up 3, out 2, up 12
$y = 3(x - 3)^2 - 7$

Page 103 - Factorised Equations 1

A1 a) $y = 2 \times {^-4} = {^-8}$
b) $x = {^-2}$ or $x = 4$
c)



d) for vertex $x = 1$,
$y = 3 \times {^-3} = {^-9}$

B1 y-int : (0, 8) ; x-int $(^-2, 0)$ $(^-4, 0)$;
vertex : $(^-3, ^-1)$

B2 y-int : $(0, ^-6)$; x-int (2, 0) $(^-3, 0)$;
vertex : $(-\frac{1}{2}, {^-6}\frac{1}{4})$

B3 a) x-int $(^-3, 0)$ (5, 0) ; vertex : $(1, ^-16)$
b) x-int (0, 0) $(^-3, 0)$; vertex : $(^-1\frac{1}{2}, -2\frac{1}{4})$

Page 104 - Factorised Equations 2

A1 a) y-int : $(0, ^-1\frac{1}{2})$; x-int $(^-3, 0)$ (1, 0) ;
vertex : $(^-1, ^-2)$
b) y-int : $(0, ^-4)$; x-int (1, 0) (2, 0) ;
vertex : $(1\frac{1}{2}, \frac{1}{2})$
c) y-int : $(0, ^-12)$; x-int (2, 0) $(^-2, 0)$;
vertex : $(0, ^-12)$

B1 a) $y = (x - 3)(x - 2)$
b) y-int (0, 6); x-int (3, 0)(2, 0)
vertex : $(2\frac{1}{2}, -\frac{1}{4})$

B2 a) $y = (x + 1)(x - 3)$
y-int $(0, ^-3)$; x-int $(^-1, 0)(3, 0)$
vertex : $(1, ^-4)$
b) $y = x(x - 2)$
y-int (0, 0); x-int (0, 0)(2, 0)
vertex : $(1, ^-1)$
c) $y = (2x + 1)(x - 3)$
y-int $(0, ^-3)$; x-int $(-\frac{1}{2}, 0)(3, 0)$
vertex : $(1\frac{1}{4}, {^-6}\frac{1}{8})$

Page 105 - Factorised Equations 3

A1 a) $y = (x + 1)(x - 2)$ b) $(0, ^-2)$
c) vertex : $(\frac{1}{2}, {^-2}\frac{1}{4})$
A2 a) $y = {^-(x + 2)(x - 4)}$ b) (0, 8)
c) vertex : (1, 9)
A3 a) $y = (x + 3)(x - 1)$ b) $(0, ^-3)$
c) vertex : $(^-1, ^-4)$
A4 a) $y = {^-(x + 2)(x - 6)}$ b) (0, 12)
c) vertex : (2, 16)
B1 a) $y = a(x + 1)(x - 3)$; $^-6 = a \times 3 \times {^-1}$ then $a = 2$
equation : $y = 2(x + 1)(x - 3)$
b) equation : $y = -\frac{1}{3}x(x + 5)$
c) equation : $y = \frac{1}{5}(x + 8)(x - 10)$

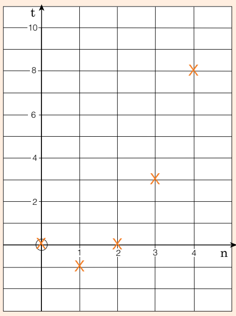
Page 106 - Writing Quadratic Equations 1

A1 a) $y = \frac{1}{2}x^2 + 1$ b) $y = \frac{1}{3}(x + 2)(x - 3)$
c) $y = {^-(x + 2)^2} + 3$ d) $y = 0.3\,x\,(x + 3)$
e) $y = -\frac{2}{3}x^2 + 10$ f) $y = 0.08\,(x - 5)^2$

Page 107 - Writing Quadratic Equations 2

A1 a) $y = 2(x + 3)(x + 1)$ $y = 2(x + 2)^2 - 2$
b) $2(x + 3)(x + 1) = 2(x^2 + 4x + 3) = 2x^2 + 8x + 6$
$2(x + 2)^2 - 2 = 2(x^2 + 4x + 4) - 2 = 2x^2 + 8x + 6$

A2 a)



b) $t = n(n - 2)$;
$t = (n - 1)^2 - 1$
$t = n^2 - 2n$
c) $t = 360$

B1 A - k B - g
C - c D - i
E - f F - d
G - b H - l
I - e J - j
K - a L - h

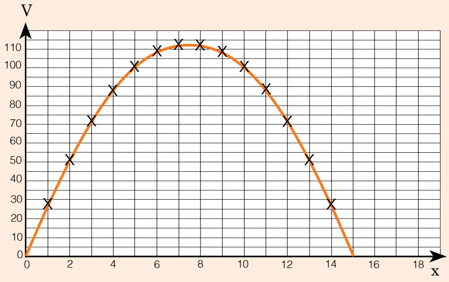
Page 108 - Optimisation Problem 1

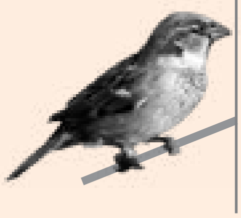
A1 If $x = 3$, then $h = 12$ and $V = 72$ cm^3

A2

x	1	2	3	4	5	6	7	8	9	. . .
h	14	13	12	11	10	9	8	7	6	. . .
V	28	52	72	88	100	108	112	112	108	. . .

For whole number values of x, the maximum volume is reached for $x = 7$ or $x = 8$, $V = 112$ m^3.
The relationship is quadratic because the second difference is $^-4$. The equation for the parabola is $y = ax(x - 15)$ and $a = {^-2}$.
So equation : $y = {^-2}x(x - 15)$. Maximum volume reached when $x = 7.5$ cm, $V = 112.5$ cm^3.





Page 109 - Optimisation Problem 2

A1 If $x = 3$, then $2 \times 3 + y = 60$ so $y = 54$,
then area is $3 \times 54 = 162$.

x	y	Area		
1	58	58		
			54	
2	56	112		$^-4$
			50	
3	54	162		$^-4$
			46	
4	52	208		$^-4$
			42	
5	50	250		

The table shows the relationship between x and area is quadratic, with second difference $^-4$, so the equation starts with $^-2x^2$.

x	A	pattern
1	58	$^-2 \times 1 + 60$
2	112	$^-2 \times 4 + 120$
3	162	$^-2 \times 9 + 180$
4	208	$^-2 \times 16 + 240$

So equation : $A = {^-2}x^2 + 60x$ or $A = {^-2}x^2(x - 30)$
Max area found when x is halfway between
0 and 30, so at $x = 15$
then $A = {^-2} \times 15 \times {^-15} = 450$.
Hence max area is 450 m^2, with $x = 15$ m, $y = 30$ m.

Page 110 - Writing Equations Using Technology

B1 a) $y = 1.5x^2 + 1$ b) 151
B2 a) $y = 2.25x + 10.35$ b) 32.85
B3 a) $y = 2.5 \times 2.2^x$ b) 6640.0 (1 dp)
B4 a) $y = 0.5x^2 + 35x + 3$ b) 403
B5 a) $y = 1000 \times 1.1^x$ b) 2593.7 (1 dp)

Page 111-113 - Practice Investigation 1

A Total Volume - 758 cm^3
Weight = 0.91 kg Resin cost
Exclusive GST + \$16.62
Stick cost Exclustive GST = \$57.60
Total Cost = \$74.22

B 12.73 cm

C 14.38 cm

Page 114-116 - Practice Investigation 2

A Cost = \$146.08

B Cost = \$18.10

C The cylinder diameter which uses the most plastic is 18cm, this surface area is 802cm 2 (0 d.p.)

Diameter	Surface Area
4	314
6	443
8	553
10	644
12	716
14	716
16	804
18	820
20	817
22	795
24	754