A Frequency and Relative Frequency

A **frequency table** can be used to summarise the data shown in a **tally chart**.
In a frequency table the first column lists the **outcomes** and the second shows the **tally**. In the third column the tally marks are counted to give the **frequencies**.

The frequency table can be extended with an extra column showing the **relative frequency** of each outcome. Relative frequency shows the proportion of times each outcome occurred.

Example : In a game, a coin is tossed and a die is rolled. A head is worth 2 points, a tail is worth 1 point and the score of each trial is found by adding the coin value with the number on the die. For instance, a result of H3 has a score of 2 + 3 = 5.
To find out the likelihood of scores in this game the coin was tossed and the die rolled 40 times. These are the results :

Toss :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Result :	H3	H1	T1	T5	H2	H5	H2	H2	T6	T5	T6	H6	T1	H3	T3	H6	H2	H5	T1	H5
Score :	5	3	2	6	4	7	4	4	7	6	7	8	2	5	4	8	4	7	2	7

Toss :	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Result :	T1	H2	T3	H4	T2	H1	T3	T3	T2	H5	T6	T4	H5	H4	H2	T6	H3	T6	H1	H4
Score :	2	4	4	6	3	3	4	4	3	7	7	5	7	6	4	7	5	7	3	6

a) Organise the data in a frequency table.   b) Calculate the relative frequency of each outcome.

Answers :
a) The lowest possible score is 2 and the highest is 8.
The scores (on the bottom line of the table) are tallied.
The **frequency** of each score is found by counting the tallies
b) The relative frequency of each score is found by dividing its frequency by the total number of tosses.
So the relative frequency of score 2 is $\frac{4}{40}$ = 0.1.

*Relative frequency is often called the **experimental probability** of each outcome. We can conclude that in this game, after 40 trials, the experimental probability of a score of 6 is 0.125; P(6) = 0.125*

score	tally	freq.	relative freq.
2	||||	4	0.1
3	ⵜ卌	5	0.125
4	卌 卌	10	0.25
5	||||	4	0.1
6	卌	5	0.125
7	卌 卌	10	0.25
8	||	2	0.05
Total		40	1

The relative frequencies should add to 1.

1   When a drawing pin is tossed on a table, it either lands with its point up (**U**) or with the point sloping down (**D**).
In an experiment three drawing pins are tossed simultaneously onto a table. The number of pins pointing upwards is counted for each toss. This is done 30 times and the table shows the results.

Toss :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Result :	UUD	UDU	DDU	UUU	DUU	DDD	UUU	UUD	UDU	DDU	UDU	DDD	DUD	UUU	UDU
Nº of Ups :	2	2	1												

Toss :	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Result :	UDD	DDD	DDU	UUD	DUU	DUD	UUU	UDD	UUU	UDU	UDU	DDU	DUU	UUU	DDD
Nº of Ups :															

a) Complete the bottom line in the table.

b) List the four possible outcomes. ………………………………

c) Tally the results and complete the frequency column.

d) Complete the relative frequency column.

e) Find the experimental probability that when three pins are dropped on a table . . .

   i)   two of these pins land point up. …………………….………

   ii) all of them land point down. ………………………………

Nº of ups	tally	freq.	rel. freq.
Total			

E **Rolling Five Dice**

When making a frequency table it is important that the result of each trial is recorded with **exactly one tally stroke**.
The result of any trial may not have two different outcomes.

1   Alan decides to complete an experiment where five ordinary dice are rolled together 60 times.
     A trial will consist of rolling the five dice and recording how many of the dice show the same number.
     For example, the set (2 2 2 4 5) gets a score of 3, because 3 of the dice show the same number.

a)   What score would Alan give : i)   (1 3 3 4 6) ………………..   ii)   (1 5 5 5 5) …………………

b)   Alan needs to make decisions on how to record the results of some trials. His decisions may differ from other students.
     For instance, Alan decides to give (1 2 4 5 6) a score of 1. His reason is : *'each number is different, it occurs only once.'*

     i)   Alan must make a decision about the outcome (4 4 4 6 6). Should it score a 3 or score a 2, or perhaps both?

          Alan decides to give it a score of 3. What could be his reason? ………………………………………………………………………

          ……..

     ii) How do you think he would record (1 1 2 2 6)? Say why. ……………………………………………………………………

          ……..

c)   Alan was told that he should have defined **a trial** as *'rolling the five dice and recording the maximum number of these dice
     showing the same number'*. Explain why this definition would be better.

     ……..

     ……..

2   You are now going to complete this experiment using the definition in c) above.

a)   List the set of possible outcomes for a trial. ………………………………………………………………………………………..

b)   What outcome do you think is the most likely? ………………………………………………………………………………………..

c)   What do you think is more likely; 3 of a kind or all numbers different? ………………………………………………………………..

3   Let the trials begin! You can collect your own data, or, if you are unable to get five dice, use Alan's results of 60 trials.

(1 2 2 4 5)	(1 1 2 6 6)	(2 3 3 3 5)	(2 2 5 5 6)	(1 3 4 4 5)	(1 1 1 4 6)	(2 3 3 6 6)	(1 3 4 5 6)	(2 3 5 6 6)
(1 3 3 4 6)	(1 2 3 4 6)	(1 3 4 5 5)	(2 2 3 4 6)	(2 3 3 3 5)	(1 2 5 5 6)	(1 1 4 4 6)	(1 2 3 5 6)	(2 3 4 4 6)
(1 1 3 5 6)	(2 2 3 3 5)	(3 3 5 6 6)	(1 1 3 4 6)	(2 2 3 5 6)	(1 1 2 5 5)	(2 2 3 3 5)	(1 1 1 6 6)	(2 2 3 6 6)
(1 1 2 4 6)	(1 2 3 5 6)	(1 2 3 3 5)	(1 3 4 5 6)	(2 3 3 4 5)	(1 3 3 3 5)	(2 4 5 5 6)	(1 3 4 5 6)	(1 4 5 6 6)
(2 2 3 6 6)	(1 2 2 4 6)	(4 4 4 4 5)	(1 2 3 5 5)	(1 3 4 4 5)	(1 2 2 3 5)	(1 3 3 4 6)	(2 3 3 4 5)	(1 2 2 3 3)
(2 2 3 4 4)	(2 3 3 3 6)	(1 5 5 6 6)	(1 2 3 4 6)	(1 2 3 3 6)	(2 3 4 5 5)	(3 3 5 5 6)	(4 5 5 5 6)	(1 1 4 4 5)
(3 3 4 5 6)	(2 4 4 5 6)	(3 3 3 4 6)	(1 1 4 4 6)	(1 2 3 4 4)	(1 4 6 6 6)			

a)   Find the experimental probability distribution for the experiment.

b)   Do your predictions of question 2 match the table?

     ………………………………………………………………………

     ………………………………………………………………………

     ………………………………………………………………………

     ………………………………………………………………………

c)   If the table shows that $P(5) = 0$, does that mean that it is
     impossible to roll 5 of the same? Explain your answer.

     ………………………………………………………………………

     ………………………………………………………………………

score	tally	frequency	experimental probability
Total			

A Tables and Graphs

For your investigation you must select and use **more than one display**, one of which must show the **experimental probability distribution**.

Example : Investigative question : If a marble is released at the top of the grid what are the chances that the marble finishes at each of the points A, B, C, D, E?

Data : The data is a series of letters that the marble ended up at each time it was released. Sixty trials of this experiment produced the following data :

B B C A E	C D B C C	C D B D B
C E D B B	C C C A C	B D B C A
B C D B D	B C C D C	D A C B D
C D B D A	C D D B D	D C C B B

Display the data for this investigation.

Possible Displays :

Display One is a tally table showing frequencies and the experimental probability distribution. (The experimental probability distribution is found by dividing each frequency by the total number of trials, in this case 60.)

Display Two could be a frequency graph or a graph of the experimental probabilities.

Alternative Display : While a frequency graph using all the outcomes is likely to be best for noticing patterns, other graphs could also be appropriate. These must relate to the investigative question.

For example if the question was *'How does the chance of finishing at A or E compare with the chance of finishing at C?'*, the frequencies for finishing at A or E and the frequency of finishing at C could be graphed.
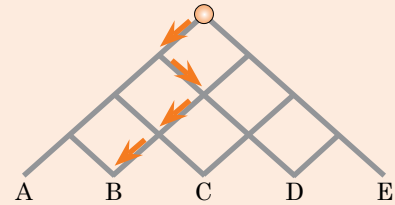
letter	tally	frequency	experimental probability
A	$\cancel{||||}$	5	0.08
B	$\cancel{||||}$ $\cancel{||||}$ $\cancel{||||}$ $||$	17	0.28
C	$\cancel{||||}$ $\cancel{||||}$ $\cancel{||||}$ $\cancel{||||}$	20	0.33
D	$\cancel{||||}$ $\cancel{||||}$ $\cancel{||||}$ $|$	16	0.27
E	$||$	2	0.03

Where the Marble Ends Up (Frequency vs Finishing Letter)

Where the Marble Ends Up (Experimental Probability vs Finishing Letter)

Comparison of the Marble Ending Up at A or E with it Ending Up at C (Frequency vs A or E / C)
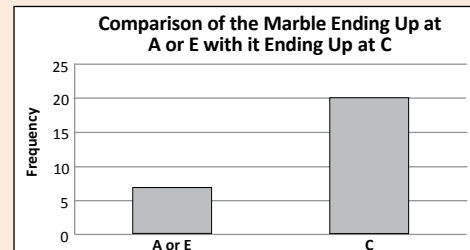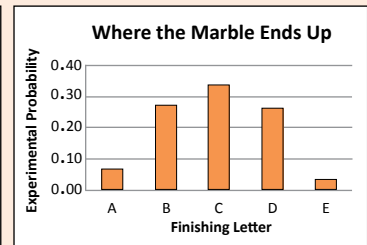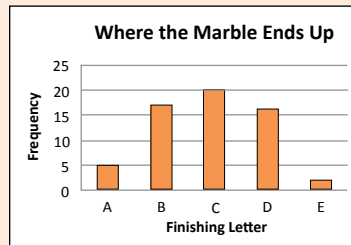
Display the data for the following investigations (remember the experimental probability distribution **must** be on one of your displays). You can either draw the graphs yourself or paste a print-out of a computer generated one in the box.

1 **Investigative Question :** What is the probability of getting each of the possible number of complete levels on a card house before it collapses?

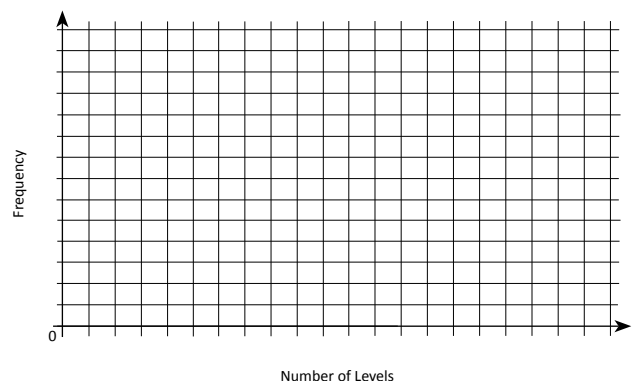**Data :** The number of complete levels for 40 trials :

1 0 1 2 3	3 0 2 4 4	4 4 3 2 0	4 4 4 5 3	2 0 5 6 4	3 5 0 4 3	0 4 3 1 3	0 0 4 3 2

Display One :

Nᵒ levels	tally	frequency	experimental probability
0			
1			
2			
3			
4			
5			
6			

Display Two :

Height of a Card House Before it Collapses

(graph: Frequency vs Number of Levels)

A Answering the Investigative Question - continued

5 Investigative Question : In ten pin bowling how does my chance of knocking over 9 or 10 pins compare with my chance of knocking only one pin over or missing all pins with one roll of a ball?

The **prediction** was that the chance of knocking over 9 or 10 pins was much greater than the chance of missing or getting only one pin.

Nº of Pins Over	Frequency	Exper. Prob.
0	3	0.03
1	5	0.04
2	14	0.12
3	23	0.19
4	10	0.08
5	8	0.07
6	6	0.05
7	18	0.15
8	21	0.18
9	8	0.07
10	4	0.03

Results of 120 Bowls at Ten Pin Bowling
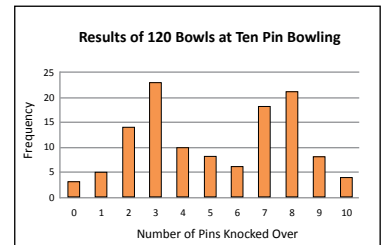
Conclusion : ………
……
……
……
……
……
……
……
……
……
……
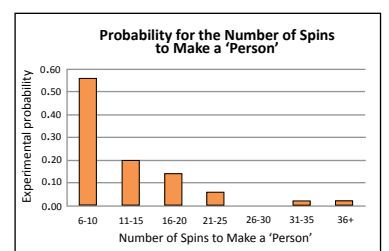
6 Investigative Question : In the *Make a Person* game what are the chances that the number of spins needed to a make a *person* is from 6 to 10, 11 to 15, 16 to 20, . . .?

The **prediction** was that the most likely number of spins would be the 11-15 grouping.

Nº Spins	Frequency	Exper. Prob.
6 - 10	28	0.56
11 -15	10	0.20
16 - 20	7	0.14
21 - 25	3	0.06
26 - 30	0	0.00
31 - 35	1	0.02
36+	1	0.02

Probability for the Number of Spins to Make a 'Person'

Conclusion : ………
……
……
……
……
……
……
……
……
……
……
……