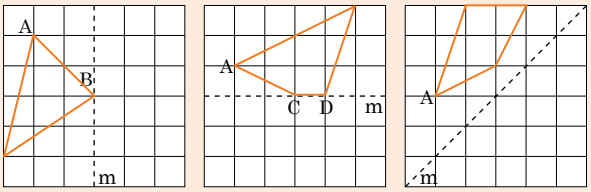
3 Reflection 1

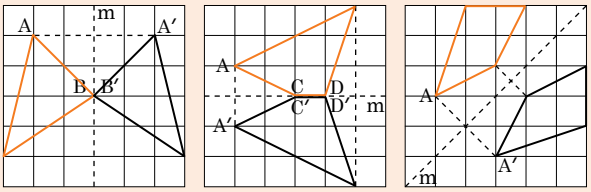
A Flip Over

A **reflection** flips the object over to the other side of a line called the **mirror line** which means that object and image are congruent.
The original point A and its image A′ are the same distance to the mirror line and line $\overline{AA'}$ is perpendicular to the mirror line.
The reflection of a polygon in mirror line m is found by flipping each vertex in turn to the other side of the line.

Examples : Draw the reflection of each object in the dotted mirror lines labelled m.



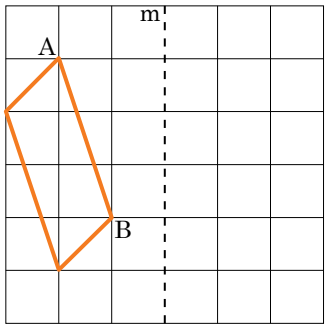
Answers :



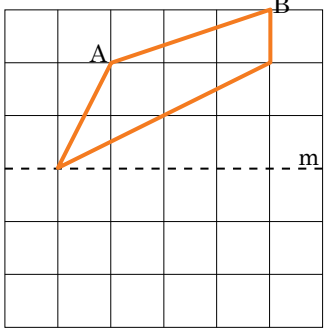
Note : If a point on the object is on the mirror line (B), then its image is that same point (B′ = B). We say B is an **invariant point** for the reflection.
Line segment $\overline{CD}$ and its image $\overline{C'D'}$ are the same, so all points on line segment $\overline{CD}$ are invariant for the reflection.

1 Draw the reflection of each object in the dotted mirror lines labelled m.

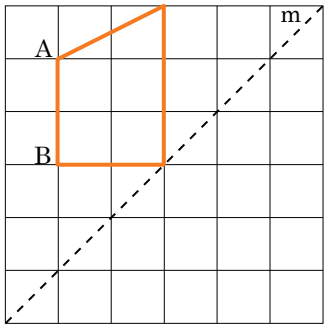
a)



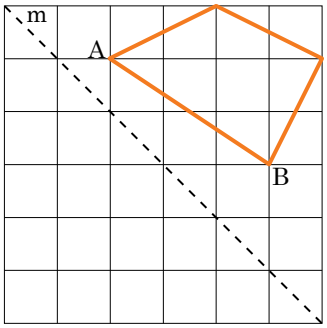
b)



c)



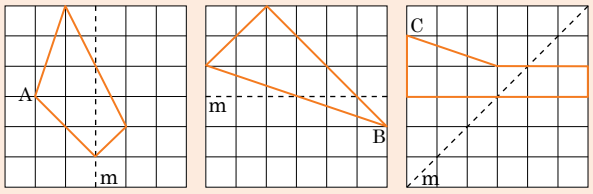
d)



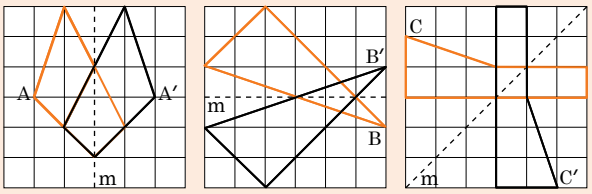
2 Label with P any invariant points you can find in the above reflections.

The shape may be *sticking through the mirror line*.

Examples : Draw the reflection of these shapes in the dotted mirror lines labelled m.

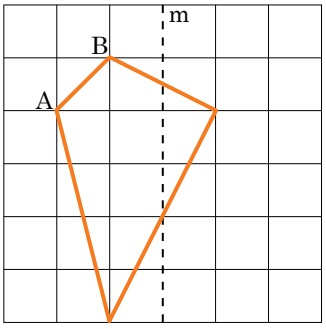


Answers :

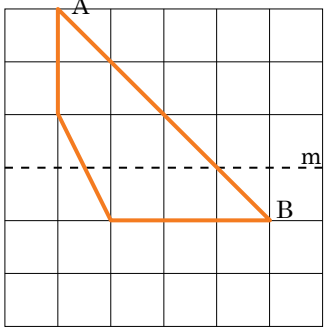


3 Draw the reflection of each shape in the dotted mirror line labelled m.

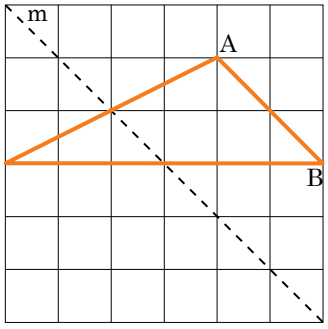
a)



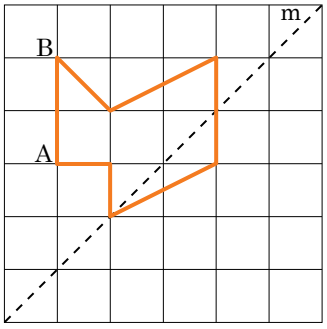
b)



c)



d)



4 Do the above reflections show invariant points? If so mark them in blue.

A Negative Scale Factor

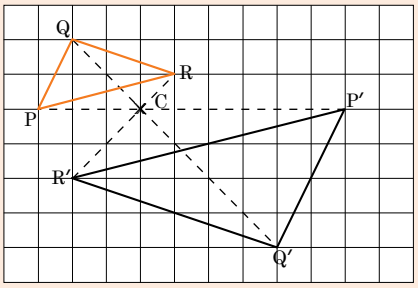
If the scale factor of an enlargement is negative, then the image is on the opposite side of the centre.

Example : Enlarge ΔPQR with $k = -2$, centre of enlargement C.

Working :

Join C to P, make $\overline{CP}$ twice as long but go the opposite direction. Repeat for Q and R.

Check that the sides of the image are twice as long as the sides in the original.

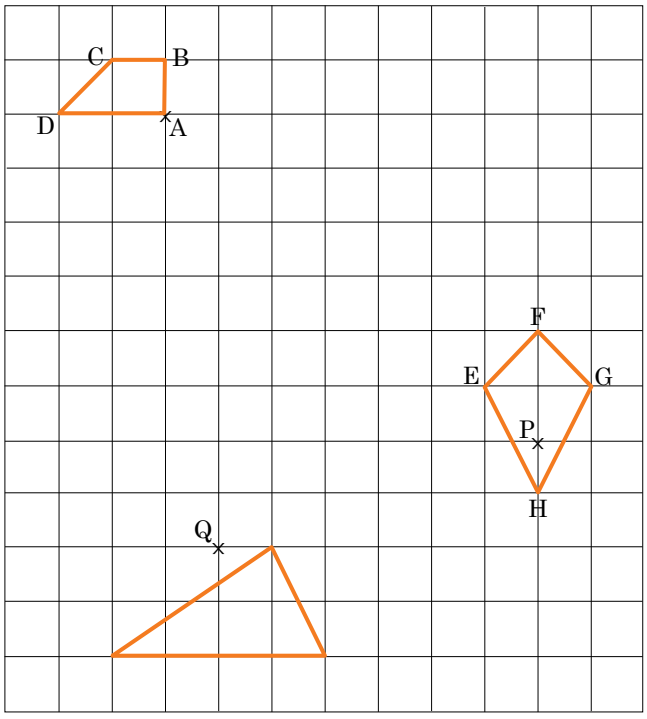


1 Enlarge with the given scale factor and centre of enlargement.

a) The trapezium with $k = -3$, centre A.

b) The kite with $k = -2$, centre P.

c) The triangle with $k = -1$, centre Q.



2 Levi looks at the result of 1 c) above and comments that it looks like another type of transformation he has done. What transformation would that be?

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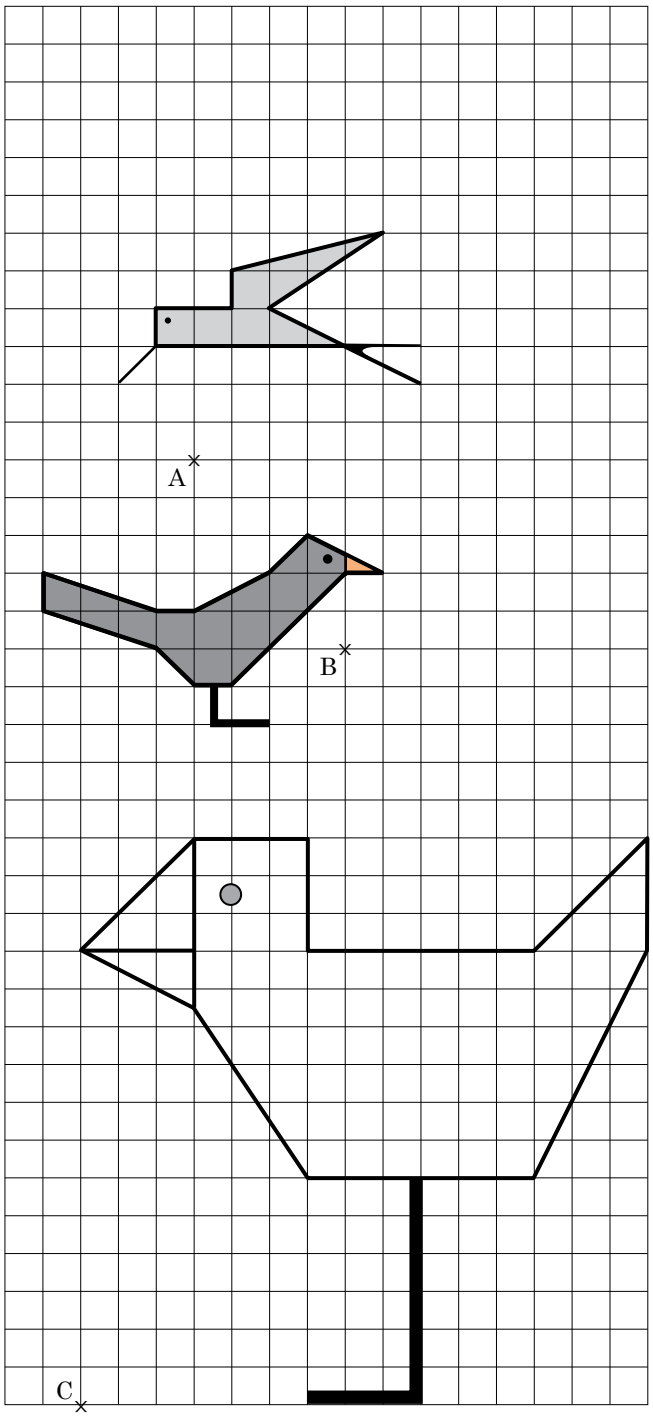
B Speedy Enlargements

You could do an enlargement faster by finding the image of just one point and use that as a starting point to draw the shape, making all edges k times as long. If k is negative you need to also rotate the shape 180°.

1a) Enlarge the swallow with centre A, scale factor 2.

b) Enlarge the black bird with centre B, scale factor -1.

c) Enlarge the pukeko chick with centre C, scale factor $\frac{2}{3}$.



15 Multiple Transformations 1

A Double Reflections Revisited

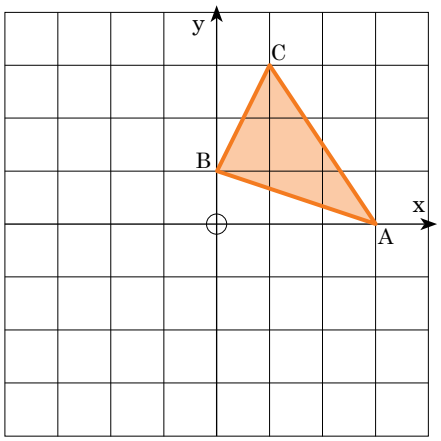
Look back to pages 2, 4, 6 and 9 where we did double translations, double reflections, double rotations and double enlargements.

On page 2 we found that the final image after two translations, (one after the other) can be found by giving the original just one single translation.

We saw on pages 6 and 9 that the final image after two rotations (or two enlargements) can be found by giving the original just one single rotation (or enlargement).

However on page 4 we found that the final image after two reflections, can't be found by giving the original just one single reflection.

1



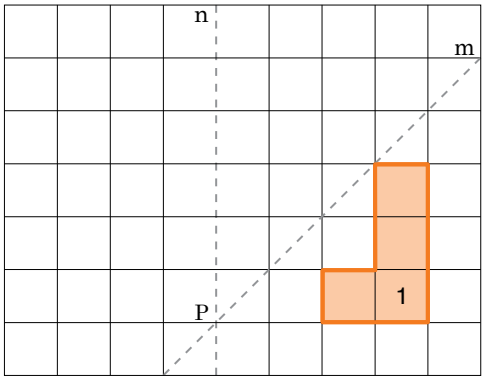
a) Reflect ΔABC in the y-axis giving $\Delta A'B'C'$.

b) Reflect $\Delta A'B'C'$ in the x-axis giving $\Delta A''B''C''$.

c) Complete : ΔABC maps straight onto $\Delta A''B''C''$ by

a rotation with centre ..... and angle ..... .

2



a) Reflect shape **1** in line m giving shape **2**.

b) Reflect shape **2** in line n giving shape **3**.

c) Complete : Shape **1** maps straight onto shape **3** by

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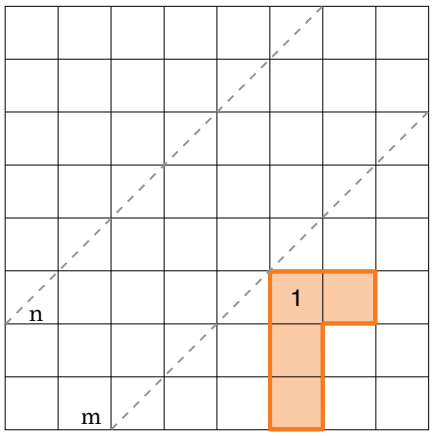
B Investigate

1



a) Reflect shape **1** into line m. Label the image **2**.

b) Reflect shape **2** into line n. Label the image **3**.

c) Describe the single transformation which maps **1** onto **3**.

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2 What if we did 3 reflections in 3 different mirror lines? Would it be possible to get the final image by giving the original just one single reflection?

Investigate using grid paper at the back of this booklet. Try many different positions of the 3 mirror lines, and then write a paragraph about your findings.

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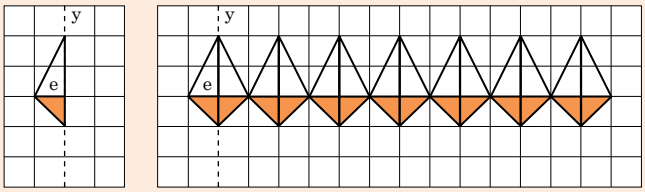
A Creating a Strip Pattern

We can make interesting patterns by using a simple shape as the object for **multiple transformations**.

Example : Reflect the given triangle (e) in the vertical line y, then give the entire shape many translations of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$*.

* this means we move the shape 2 to the right, and again 2 to the right, until the pattern becomes clear.



In the exercises below the same simple triangle will undergo a series of transformations to make different strip patterns. Carefully follow the instructions.

1a) Reflect triangle **1** in the vertical mirror line y.

b) Give the new shape a glide reflection with horizontal mirror line x and vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

c) Give the entire shape many translations of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

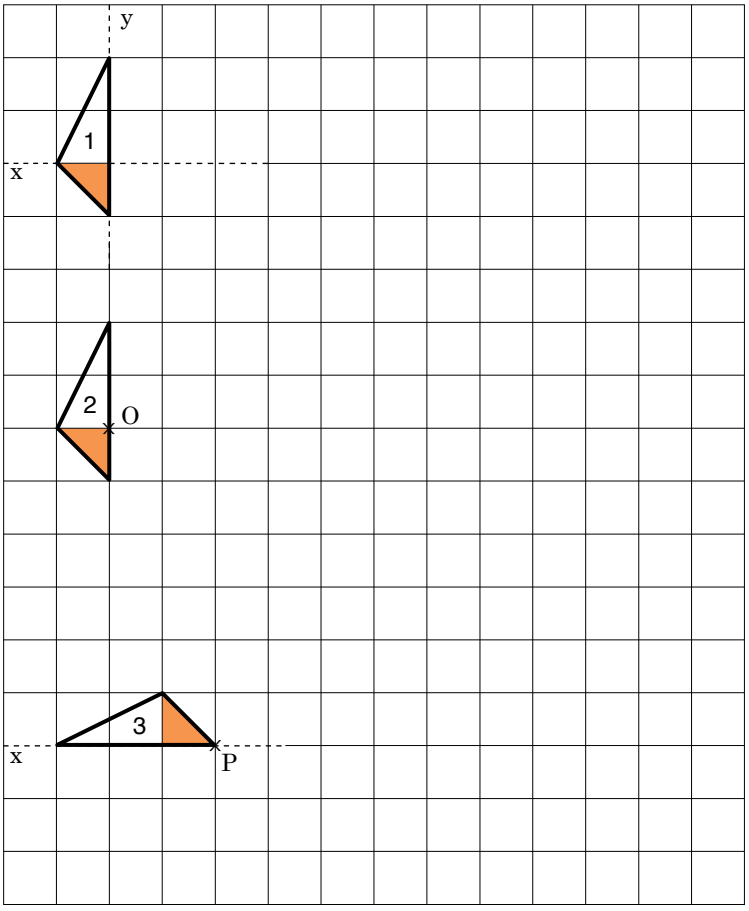
2a) Rotate triangle **2** 180° about centre O.

b) Give the entire shape many translations of $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

3a) Rotate triangle **3** 270° about centre P.

b) Reflect the two triangles in horizontal line x.

c) Give the entire shape many translations of $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.



4a) Enlarge the triangle with centre Q and k = 2.

b) Give the combined shape a glide reflection with horizontal mirror line x and vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

c) Give the entire shape many translations of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.