4 Basic Facts 1

A Place Values

Our counting system is based on ten digits, 0 to 9. Ten ones make a ten, ten tens make a hundred, etc. This diagram illustrates the way we name numbers.

Billions			Millions			Thousands					
H	T	O	H	T	O	H	T	O	H	T	O

Examples : Consider the number 58109320000
a) Write down the place value of the digit 3.
b) Write the number in words.

Working : Group the numbers from the right in groups of three :

B	M	T	
5 8	1 0 9	3 2 0	0 0 0

a) the digit 3 has place value hundred thousand.
b) fifty-eight billion, one hundred and nine million, three hundred and twenty thousand.

1 In 2022 the world population was about 7 968 900 000.

a) What is the place value of the digit 6?

....

b) Write the number in words. ....

....

....

....

2 Write these numbers in digits :

a) three hundred and twenty million, five thousand, one hundred and fifteen.

....

b) forty billion, six million, three hundred thousand and twenty.

....

B Millions of Dollars

Examples : a) A suitcase holds thirty-five thousand bills of $100. How much money is that?

b) $4 653 500 in cash is being bundled in lots of $10 000. How many bundles will there be? How much money is left?

Working: a) Imagine 35 000 in the frame, with the last digit in the hundreds position. Read off the answer.

3	5	0	0	0	0	0

Answer : $3 500 000

b) Imagine 4653500 in the frame.

4	6	5	3	5	0	0

Read off the number which has its last digit in the ten thousand position.

Answer : 465 bundles of $10 000, $3500 left.

1 A bank vault holds five hundred and twenty seven bundles of ten thousand dollars. How many dollars is this?

....

2 $248 715 is paid out in bundles of one thousand dollars. How many bundles are there?
How much is left?

....

....

....

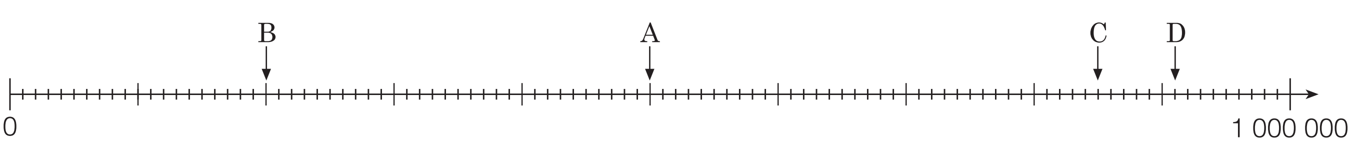
....

3 Calculate : 41300 x 1000

....

C Number Line

1



a) The numberline goes from zero to one million. Pointer A is halfway to one million. A is at ....

b) Write the number shown by each arrow. B is at ...., C is at ...., D is at ....

c) Draw arrows P, Q and R to show the place of these numbers. P is at 100 000, Q is at 750 000, R is at 630 000.

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20 Prime Numbers

Number Facts

A Primes, Prime Factors

A **prime number** has just two factors, namely 1 and itself.

Example : Explain why 13 is a prime, but 14 is not.

Answer : 13 = 1 x 13 factors 1, 13
14 = 1 x 14, 2 x 7 factors 1, 2, 7, 14
Since 13 has two factors, it is a prime.
Since 14 has more than two factors, it is not a prime.

1 Circle the prime numbers in this box.

11	13	15	17	19
21	23	25	27	29

2 Explain why 1 is not a prime number.

....

....

3 There is only one number which is a prime number *and* an even number. Which number? ....

4 List all prime numbers . . .

a) under 10 : ....

b) between 30 and 40 : ....

Example : List all **prime factors** of 56

Working : The factors of 56 are 1, 2, 4, 7, 8, 14, 28, 56.
Of these factors, 2 and 7 are primes.

Answer : Prime factors of 56 are 2 and 7.

5 List all prime factors of . . .

a) 44 : ....

....

....

b) 45 : ....

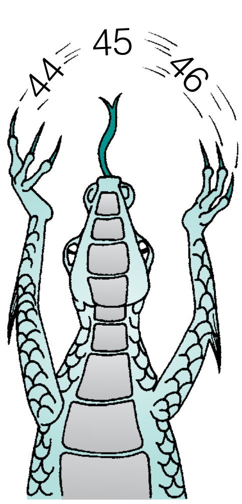
....

....

c) 46 : ....

....

....



B Product of Prime Factors

Product is another word for multiplication.

Examples :
a) Write 56 as a product.
b) Write 56 as a product of prime factors.

Working :
a) 56 = 8 x 7
b) In the above product, 7 is prime, 8 is not prime.
We continue to write products for numbers that are not primes.
$56 = 8 \times 7 = 2 \times 4 \times 7 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$

1 Write as a product of prime factors :

a) 81 = ...

b) 76 = ...

....

c) 100 = ...

....

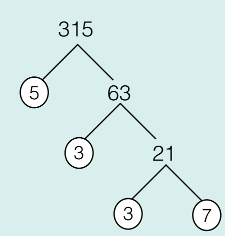
d) 48 = ...

....

A **factor tree** is a tool to break down a number to its prime factors.

Example : Draw a factor tree and write 315 as a product of prime factors.

Answer : 315 = 7 x 5 x 3 x 3
$= 7 \times 5 \times 3^2$



2 Draw factor trees and write the numbers as a product of prime factors.

a) 180

b) 216

180 = ...

216 = ...

Rounding Decimals 35

When doing calculations on a calculator we could end up with a screen full of digits. If it is not sensible to write an answer which is too precise then we can round the answer, showing only a few digits.

A Rounding Small Numbers

If we are **rounding to the nearest whole number**, then the *tenths* digit is used to decide whether to round up or down.

Examples : Round these numbers to the nearest whole number.
a) 2.4 b) 18.5 c) 6.38

Working : Imagine you are counting in ones.

a) 2.4 is between 2 and 3, but closer to 2.
2.4 = 2 (to the nearest whole number)

b) 18.5 is exactly halfway between 18 and 19, we round up.
18.5 = 19 (to the nearest whole number)

c) 6.38 = 6 (to the nearest whole number)

1 Round these to the nearest whole number.

a) 6.8 .... b) 14.3 ....

c) 27.5 .... d) 43.74 ....

e) 128.49 .... f) 899.8 ....

If we are **rounding to the nearest tenth**, then the *hundredths* digit is used to decide whether to round up or down.
Rounding to the nearest tenth is usually described as **rounding to 1 decimal place** or in short, **1 dp.**

Examples : Round these numbers to one decimal place.
a) 4.37 b) 6.953

Working : Imagine you are counting in tenths.

a) 4.37 is between 4.3 and 4.4, closer to 4.4.
So 4.37 = 4.4 (to 1 dp)

b) 6.953 is between 6.9 and 7.0, closer to 7.0.
So 6.953 = 7.0 (to 1 dp)

2 Round these to one decimal place.

a) 7.48 .... b) 3.23 ....

c) 0.754 .... d) 9.39 ....

e) 25.03 .... f) 43.92 ....

3 Calculate on your calculator. Then round as indicated.

a) 8.7 x 3.492 = ...(to 1 dp)

b) $\frac{278}{6.6}$ = ...(nearest whole)

c) 6.325 - 0.989 = ... (to 2 dp)

B Sensible Rounding

Guidelines for **sensible rounding** :

- Round to 2 dp when the answer is an amount of money.
- Round to nearest whole when the answer is a number of items.
- When the answer is a length, weight or distance, make sure the answer makes sense.

Examples : A bag with 17 oranges weighs 3.5 kg and costs $9.28
a) What do these oranges cost per kg?
b) How many oranges in 1 kg?
c) Find the weight of 1 orange.

Working :

a) $\boxed{9.28}$ $\boxed{\div}$ $\boxed{3.5}$ $\boxed{=}$ 2.651428 Answer : $2.65 per kg.

b) $\boxed{17}$ $\boxed{\div}$ $\boxed{3.5}$ $\boxed{=}$ 4.857143 Answer : about 5 per kg.

c) $\boxed{3.5}$ $\boxed{\div}$ $\boxed{17}$ $\boxed{=}$ 0.205882 Acceptable answers :
0.2 kg (1 dp) per orange
or 0.21 kg (2 dp) per orange

1 Eddy used 24 litres of petrol to drive a distance of 256 kilometres in his car. Calculate how far the car goes on 1 litre of petrol. Round sensibly.

....

....

2 A bag with 10 kiwifruit weighs 1.45 kg.
How many kiwifruit in 1 kg? Round sensibly.

....

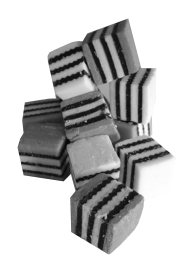
....

3 Sandra's steps are 0.68 metres long. She measured the distance from home to school to be 1048 steps. Calculate the distance from Sandra's home to school. Think carefully when rounding your answer.

....

....

4 A bag with 0.28 kg of liquorice allsorts costs $4.50. How much do the liquorice allsorts cost per kg?



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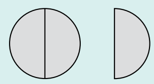
44 Mixed Numbers

A Part Whole, Part Fraction

Vocabulary : If the numerator of a fraction is larger than the denominator, then its value is larger than 1. We call such a fraction an **improper fraction**.

For instance, $\frac{3}{2}$ is an improper fraction, three halves equal one and a half.



The number $1\frac{1}{2}$ is called a **mixed number**, because it is part whole number, part **proper fraction**.

1 Write as a mixed number.

a) Fifteen halves = ...

b) Eleven quarters = ...

c) Twenty eights = ...

2 How many halves in

a) 6 ? ....   b) $10\frac{1}{2}$? ....

3 How many quarters in

a) 5 ? ....   b) $2\frac{3}{4}$? ....

4 Complete

	improper fraction	=	mixed number
a)	$\frac{11}{3}$	=	....
b)	$\frac{29}{4}$	=	....
c)	$\frac{45}{5}$	=	....
d)	$\frac{68}{6}$	=	....

5 Complete

	improper fraction	=	mixed number
a)	....	=	$2\frac{1}{2}$
b)	....	=	$3\frac{4}{5}$
c)	....	=	$7\frac{2}{3}$
d)	....	=	$10\frac{3}{4}$

6 Write decimals as mixed numbers.

	decimal	=	mixed number	=	simplified
a)	3.5	=	$3\frac{5}{10}$	=	$3\frac{1}{2}$
b)	4.08	=	....	=	....
c)	6.25	=	....	=	....
d)	1.05	=	....	=	....
e)	23.6	=	....	=	....

B Dividing Made Simpler

- In the Whole Numbers Chapter we wrote the division $23 \div 4$ as $\frac{23}{4}$ and gave the answer as 5 remainder 3.
- In the Decimal Chapter we wrote 23 as 23.00 and continued the division, leading to $\frac{23}{4} = 5.75$.

$$4\overline{)23.00}\quad 5.75$$

- By viewing the division as an improper fraction we get $\frac{23}{4} = 5\frac{3}{4}$

1 Complete

	division	=	improper fraction	=	mixed number
a)	$26 \div 5$	=	$\frac{26}{5}$	=	....
b)	$37 \div 8$	=	....	=	....
c)	$75 \div 9$	=	....	=	....
d)	$80 \div 7$	=	....	=	....
e)	$143 \div 20$	=	....	=	....
f)	$100 \div 12$	=	....	=	....

Since a division can be viewed as a fraction, we can simplify divisions the same way we simplify fractions.

Example : Divide $\frac{240}{35}$

Working : Simplify by dividing numerator and denominator by 5 :

$$\frac{240}{35} = \frac{48}{7} = 6\frac{6}{7}$$

2 Divide, giving your answer as a mixed number.

a) $\frac{500}{80}$ = ...

b) $\frac{320}{30}$ = ...

c) $\frac{64}{28}$ = ...

d) $\frac{70}{16}$ = ...

e) $\frac{200}{45}$ = ...

f) $\frac{1500}{60}$ = ...

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54 Number Patterns

A Discover the Rules

1 Study these sequences, each has their own special rule. When you discover the rule, continue the sequence with three more numbers.

a) 5, 11, 17, 23, 29, ..., ..., ...

b) 60, 57, 54, 51, 48, ..., ..., ...

c) 1, 2, 4, 8, 16, ..., ..., ...

d) 7, 6, 9, 8, 11, ..., ..., ...

2 Fill in the missing numbers in each sequence.

a) 120, 60, 30, ..., 7.5, ..., 1.875

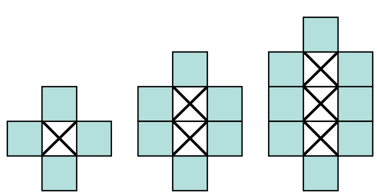
b) 1, 3, 9, ..., 81, ..., 729

c) 7, 1, $^{-}5$, ..., $^{-}17$, ..., $^{-}29$

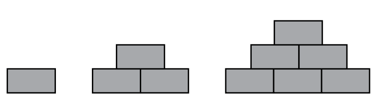
d) 1, 4, 9, ..., 25, ..., 49

3 Draw the next diagram.

a)



b)



4 For each sequence, find the next two numbers and explain how you did it.

a) 50 000, 5000, 500, 50, ..., ...

....

....

....

b) 1, 2, 4, 7, 11, ..., ...

....

....

....

B Follow the Rules

The numbers in a sequence are called **terms.**
For instance in the sequence 6, 1, $^{-}4$, $^{-}9$, . . . the first term is 6, the second term is 1, etc.
The rule for the sequence is *'the first term is 6, each term is 5 less than the previous term.'*

1 Use these rules to write down 5 terms of each sequence.

a) The first term is 8, each term is 3 less than the previous term.

............, ..., ..., ..., ...

b) The first term is 64, each term is half the previous term.

............, ..., ..., ..., ...

c) The first term is $^{-}12$, each term is 5 more than the previous term.

............, ..., ..., ..., ...

2 *The number of rabbits on a farm doubles every month, the first month there were ten.*

a) Complete the table for this rule :

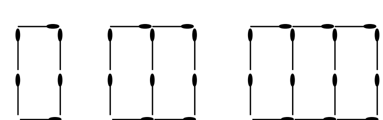
months	1	2	3	4	5	6
rabbits	***10***					

b) How many rabbits will there be after a year?

....

3 Matches are used to form a string of rectangles.

a) Draw a string with 4 rectangles.



b) Complete this rule :

The first rectangle in a string needs .... matches,

each following rectangle needs .... extra matches.

c) How many matches are needed for a string of

10 rectangles? ....

....

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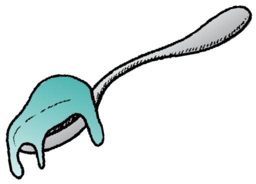
A The Green Blob!

1 

A blob of green slime sits in a laboratory dish. It occupies $\frac{1}{32}$ of the dish. During the night it suddenly starts to grow. Each hour the area occupied by the green blob doubles. How long before the whole dish is filled with green slime?

....

....

B Great Grandparents

1 Every human being has two biological parents and four grand parents who pass on their DNA. Show how you could estimate the number of people that passed on their DNA to you since the year 1500.

Do you think your estimation is accurate? Explain.

(Hint : Working backwards you could call yourself generation 0, your parents generation 1 and your grandparents generation 2 and so on.)

....

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72 Rates

A Getting Started

Things like cost, speed and rainfall are **rates**.
The unit for a rate contains the word *'per'*.

For instance : The unit for cost is dollars *per* item.
The unit for speed is km *per* hour.
The unit for rainfall is mm *per* year.

1 Match the rate with the unit.

Rate	Unit
speed	kilowatts per hour
growth rate	kilometres per hour
fuel consumption	kilometres per litre
paint coverage	dollars per pound
electricity use	centimetres per year
exchange rate	square metres per litre

2 Think of a possible unit for these rates.

a) weight loss .... per ....

b) heart rate .... per ....

c) pay rate .... per ....

d) mail sorting .... per ....

3 Jason gets paid \$5.50 per hour to baby-sit his younger sister. How much does he get paid for $3\frac{1}{2}$ hours of baby-sitting? ....

....

4 On a paint tin is printed *1 L covers 15 square metres.* How many square metres can be covered with 2.5 L?

....

5 Our car used a full tank of petrol (40 L) to drive 520 km from Hamilton to Wellington. Calculate our car's fuel consumption in kilometres per litre.

....

6a) Mr Watson exchanged 250 NZ dollars for 17500 yen. At this rate, how many yen do you get for one NZ dollar?

....

b) How many yen does Lucy Watson get for \$40?

....

B Speed

Example :

A tourist bus takes 6 hours to travel from Christchurch to Dunedin, which is a distance of 372 km.

a) What is the average speed of the bus?
b) At this speed, how far can the bus be expected to travel in 15 minutes?

Working :

a) 372 km in 6 hours, that is $372 \div 6 = 62$ km in 1 hour.
Speed = 62 km/hr

b) 15 minutes is $\frac{1}{4}$ of an hour.
distance travelled = $\frac{1}{4}$ of 62 km = 15.5 km.

1 A plane takes 5 hours to fly 4620 km.

a) What is the speed of the plane? ....

b) At this speed, how far does the plane fly in 20 minutes.

....

2 It took Rewi half an hour to bike 8 km.
What was Rewi's speed in km per hour?

....

3 At an average speed of 100 km/hr, what distance would be covered in . . .

a) 15 minutes? ....

b) 6 minutes? ....

4 Driving at an average speed of 80 km/hr how long will it take us to drive 520 km from Hamilton to Wellington?

....

....

5 Kara took 20 minutes to walk the 2 km to Emma's house.

a) If Kara kept walking at the same speed, how far would she have walked in 1 hour?

.... km

b) Kara ran back home, covering the 2 km in 15 mins. What was Kara's speed on the way home?

.... km/hr

86 Classes of Shapes

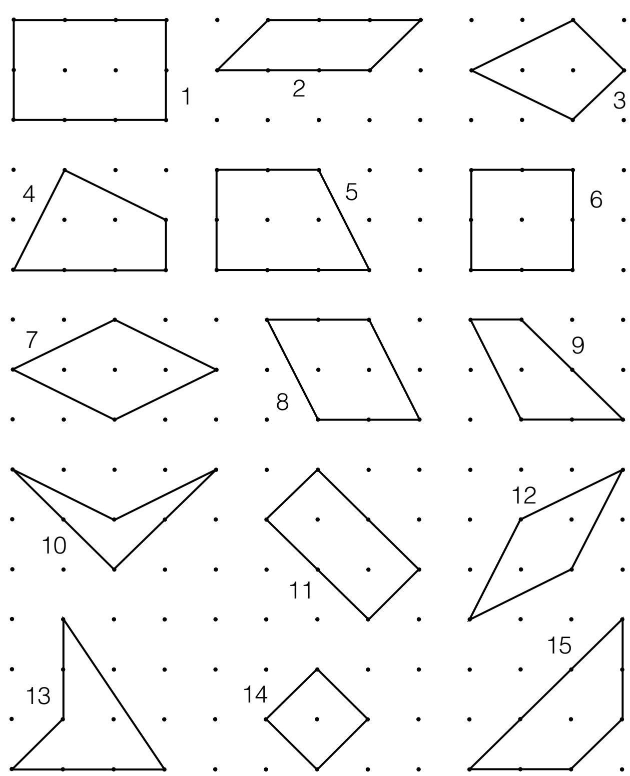
Geometry

A Classifying Quadrilaterals

Polygons with 4 sides are in a class called quadrilaterals.
This list describes 6 **sub-classes** of quadrilaterals :

Square :	all sides are equal and all angles are 90˚.
Rectangle :	two pairs of equal, parallel sides, all angles 90˚.
Parallelogram :	two pairs of equal, parallel sides.
Rhombus :	all sides equal (two pairs of parallel sides).
Kite :	two pairs of equal, adjacent sides.
Trapezium :	a pair of parallel sides.

1 Below are pictures of different quadrilaterals drawn on square dotted paper. Study each shape and find all possible subclasses it belongs to.
For instance the first quadrilateral belongs to the rectangles but also the parallelograms and the trapezia because it fits their descriptions as well.



a) squares ....

b) rectangles 1..

c) parallelograms 1..

d) rhombuses ....

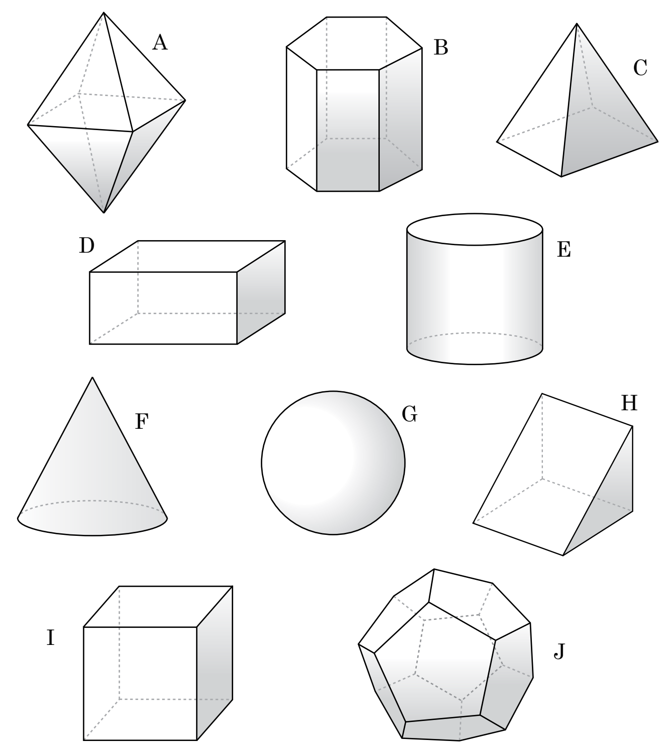
e) kites ....

f) trapeziums 1..

g) no special class ....

B Selections of Solids

1



1a) From this set of solids (A - J) select all solids which have at least one triangular face.

....

b) Select all solids with a uniform cross section.

....

2 Brittany made up a rule and said that only solids A, I and J complied with the rule. What could be Brittany's rule?

....

....

....

....

3 Solids B, C, D, H, and I comply with Mark's rule.
What could Mark's rule be?

....

....

....

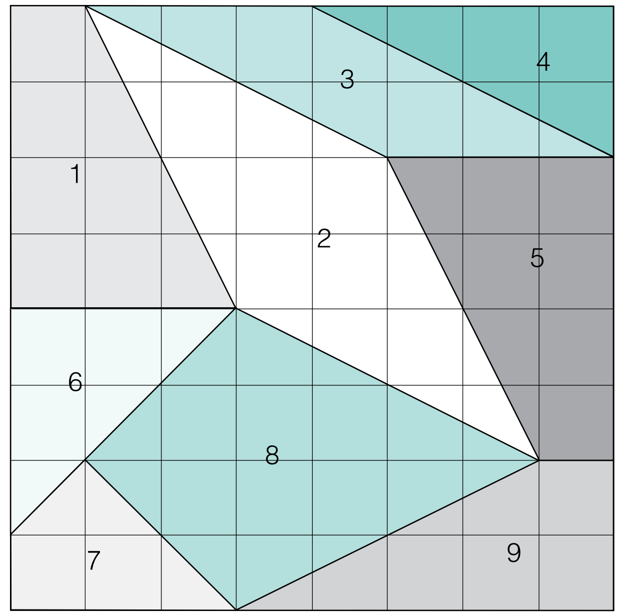
....

100 Task 3

A Polygons

1) Nine polygons are painted on a square tile. Study the shapes.

- Are they all different or are some of the shapes congruent? Write about each polygon. For instance :
- What is its most precise name?
- Work out its area and perimeter
- Is it a symmetrical shape?



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A New Zealand Households

1 Garth has used a computer spreadsheet to make this pie graph. It shows how the average New Zealand household uses electricity. The pie is drawn in 3D style, which is not always a good idea because it tends to make some slices look bigger than they ought to.

a) Which categories in the graph look bigger than they should?

Explain your choice. ....

....

....

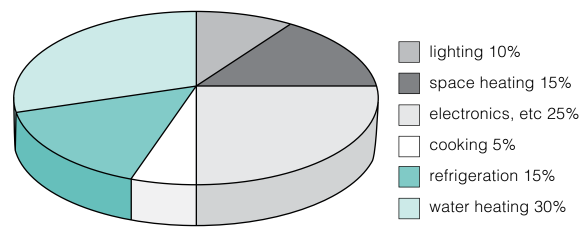
....

....

....

....

Electricity Use in the Average NZ Household



b) Redraw the pie graph in this circle, representing each category fairly. Hint : First calculate 5% of 360 degrees and then use that to calculate all the angles at the centre of the pie.

....

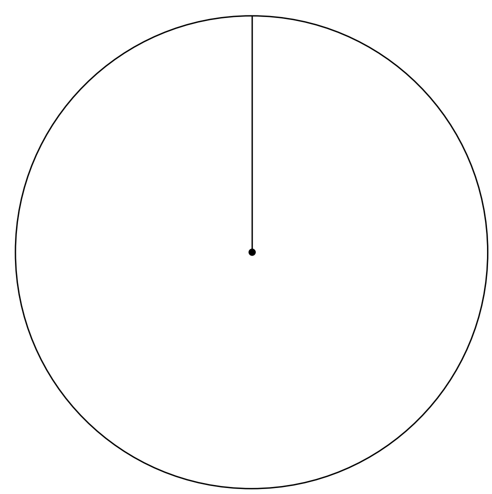
....

....

....

....

....



2 In 2016 households spent on average $220 per week on food. Money was spent in five food subgroups as follows : Fruit and vegetables $24, meat, poultry and fish $30, grocery food $92, beverages (non-alcoholic) $12, eating out and take-a-ways $62 (source : Statistics New Zealand).

a) Garth decides to make a strip graph to illustrate this information and he makes the strip 110 mm long. Finish the graph.

Average Weekly Food Expenses per Household

b) Do you think that the above graph based on data from 2016 is still useful? Explain your opinion.

....

....

....

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A7 Answer Section

Page 84 - 3D Shapes

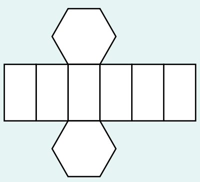
A1 2-dimensional shapes :
rectangle, triangle, circle, square, pentagon
3-dimensional shapes :
cube, pyramid, cone, cylinder, sphere, cuboid, prism

A2 a) pyramid b) prism c) cuboid d) cube

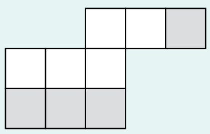
A3 A cube is a 3-dimensional shape with 6 faces, 12 edges and 8 vertices.

B1 a) hexagon b) rectangles
c) The prism has 8 faces, 18 edges, 12 vertices

B2

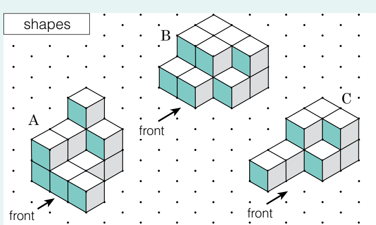


B3 There are 4 possible positions for the last square. Check that you drew just **one** of these grey squares.



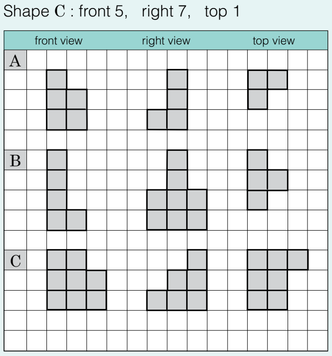
Page 85 - Views

A1

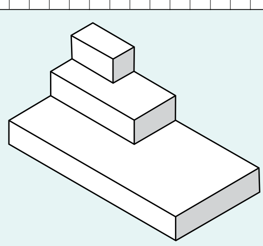


A2 Shape A : front 8, right 3, top 4
Shape B : front 9, right 2, top 6
Shape C : front 5, right 7, top 1

B1



B2



Page 86 - Classes of Shapes

A1 a) squares - 6, 14
b) rectangles - 1, 6, 11, 14
c) parallelograms - 1, 2, 6, 7, 8, 11, 12, 14
d) rhombuses - 6, 7, 12, 14
e) kites - 3, 6, 7, 10, 12, 14
f) trapeziums - 1, 2, 5, 6, 7, 8, 9, 11, 12, 14, 15
g) no special class - 4, 13

B1 a) A, C, H b) B, D, E, H, I

B2 *"All its faces have the same shape."*
(All shapes are congruent)

B3 *"At least one of the faces is rectangular."*

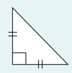
Page 87 - Angles in Ploygons

A1 obtuse, straight, right, acute, reflex

A2 a) equilateral b) isosceles

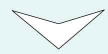
A3 a) the angles do not all have the same size.
b) obtuse = $\angle FAB$, $\angle BCD$ acute = $\angle AFE$, $\angle EDC$
right = $\angle ABC$ reflex = $\angle FED$
c) AB and FE; BC and ED; AF and CD

B1 a) e.g.



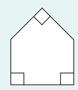
b) not possible

c) e.g.

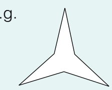


d) not possible

e) e.g.



f) e.g.



Page 88 - Angle Rules

A1 student's own estimates
measurement - top to bottom 100˚, 92˚, 17˚, 43˚

A2 225˚

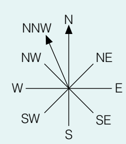
A3 a) top angle, then clockwise : 128˚, 52˚, 128˚, 52˚
b) right 66˚, left 114˚
c) top angle, then clockwise : 88˚, 143˚, 129˚

B1 size : 130˚, 135˚, 120˚, 115˚, 72˚
rule : 1, 3, 2, 3, 2

B2 angle a = 52˚, angle b = 128˚, angle c = 38˚

Page 89 - Compass Directions

A1 a) & b)



c) 45˚ + 22.5˚ = 67.5˚

A2 a) S33˚W b) N85˚E c) N72˚E d) S80˚E
e) S50˚W f) N30˚E

B1 a) N25˚W b) S10˚W c) S45˚E

B2 a) 80˚, 38˚, 38˚
b) Landmark A - Tim's answer correct
Landmark B - S80˚E,
Landmark C - Tim's answer correct
Landmark D - N38˚W

Page 90 - Bearings from North

A1 a) Te Awamutu b) 060˚
c) Huntly d) 340˚

A2 a) bearings 090˚, 180˚, 315˚, 135˚
b) bearings 240˚, 020˚, 277˚, 138˚

B1 a) at the shop b) 300 m, 015˚

B2 c) Leg 3 : 10 km, bearing 028˚

Page 91 - Constructions

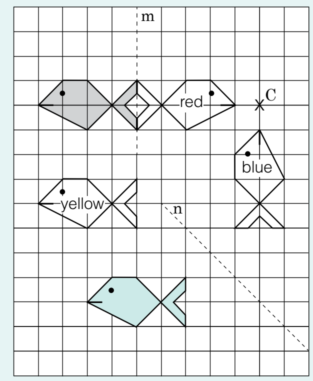
A1 - A 6 check your work with the instructions.

B1 - B4 check constructions with the sketches.

Page 92 - Transformations

A1 a)

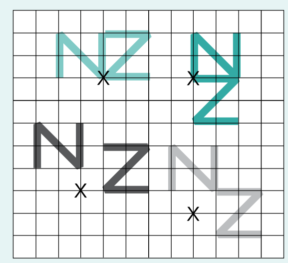
A2 a) & b)



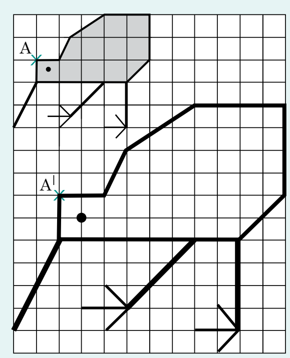
A1 b) 2 squares right, 8 squares down.

A3 A rotation with centre C over a 90˚ angle in a clockwise direction.

B1

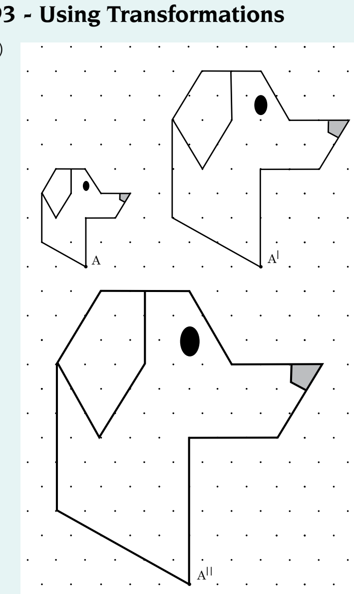


B2



Page 93 - Using Transformations

A1 a) & b)



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