Integers, Powers and Roots 3 6

A Negative Powers

1 Complete these tables and use them to understand the meaning of negative powers.

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
16	**8**					

÷ 2

3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}
81	**27**				$\frac{1}{3}$	

÷ 3

2 Write these negative powers as a fraction and a decimal.

power	fraction	decimal
2^{-1}	$\frac{1}{2}$	0.5
2^{-2}		
2^{-3}		
5^{-1}		
5^{-2}		
10^{-1}		
10^{-3}		

3 Find.

a) 5^0 ..... b) 10^0 .....

4a) If n is any whole number, what is *"n to the power of zero?"*

.....

b) Write n^{-3} as a fraction. .....

5 Use the fact that $2^{12} = 4096$ to calculate 2^{-11}.

.....

6 Calculate.

a) 2^{-5} .....

b) 4^{-4} .....

B Extension Exercise

We know : $a^2 = a \times a$ and $a^{-2} = \frac{1}{a \times a}$,
but what is meant with $a^{\frac{1}{2}}$, $a^{\frac{1}{3}}$, $a^{\frac{1}{4}}$, etc?

Answer : $a^{\frac{1}{2}} = \sqrt{a}$, $a^{\frac{1}{3}} = \sqrt[3]{a}$, $a^{\frac{1}{4}} = \sqrt[4]{a}$

Mentally calculating fractional powers is done the same way as calculating roots : use *guess and check*.

For example : $81^{\frac{1}{2}} = \sqrt{81} = 9$, because $9^2 = 81$
$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$, because $4^3 = 64$
$625^{\frac{1}{4}} = \sqrt[4]{625} = 5$, because $5^4 = 625$

1 Calculate mentally.

a) $4^{\frac{1}{2}}$ .....

b) $1000^{\frac{1}{3}}$ .....

c) $16^{\frac{1}{4}}$ .....

d) $1^{\frac{1}{6}}$ .....

e) $32^{\frac{1}{5}}$ .....

f) $(\frac{1}{4})^{\frac{1}{2}}$ .....

The [^] key on your calculator can be used to calculate any powers, positive, negative or fractional. For negative powers use the negative key [(-)], for fractional powers use the [▯/▯] key, remember to put brackets around the power.

For example : $125^{\frac{2}{3}}$ is keyed in as . . .

[125] [^] [(] [2] [▯/▯] [3] [)] [EXE],
Answer : 25

2 Check your answers to Q1 with a calculator.

3 With your calculator work out . . .

a) $16^{\frac{3}{4}}$ ..... b) $9^{\frac{3}{2}}$ .....

4 Use your answers to Q3 above to explain the meaning of . . .

a) $a^{\frac{3}{4}}$ .....

b) $a^{\frac{3}{2}}$ .....

5 Calculate with or without your calculator.

a) $27^{\frac{2}{3}}$ .....

b) $64^{\frac{1}{6}}$ .....

c) $8^{\frac{4}{3}}$ .....

17 Working with Percentages 3

A Increase/Decrease in Two Steps

Increase (or raise) means make larger.
Decrease (or reduce) means make smaller.

Example : Reduce 350 mL by 12%.

Working : 12% of 350 = 0.12 x 350 = 42 mL
350 mL – 42 mL = 308 mL

1a) Raise 180 g by 8%. .....

.....

b) Increase $4.80 by 35%. .....

.....

c) Reduce 135 kg by 42%. .....

.....

d) Decrease 2.5 m by 16%. .....

.....

2 A car decreases in value by 18% each year. The car is worth $15 000 now.

a) Calculate the value of the car next year.

.....

.....

b) What will be the value of the car the year after next?

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3 Yesterday Selina paid $165.00 for 55 litres of petrol. Overnight the price of petrol increased by 2.5%.
How much would Selina's brother pay when he fills his car with 40 litres of the same petrol today?

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B Increase/Decrease in One Step

Example : Uncle Sam's weight decreased by 8%. He used to weigh 110 kg. How much does he weigh now?

Explanation : Let w be the original weight.
New weight = w – 8% of w
= 1 x w – 0.08 x w
= (1 – 0.08) x w

Calculation : New weight = 0.92 x 110 kg = 101.2 kg

1 Show your calculation, then use your calculator.

a) Increase $560 by 18%

(1 + 0.18) x $560 = ...

b) Increase 7.5 kg by 3%

.....

c) Decrease 88.5 m by 56%

(1 – 0.56) x 88.5 m = ...

d) Decrease 320 000 by 2%

.....

2a) If gross wages are $560 and tax 20%, what are the net wages?

.....

b) If cost price is $3.85 and mark-up 45%, what is the selling price? .....

3a) A camera is priced at $280 before GST of 15% is added. How much is the price inclusive of GST?

.....

b) The camera mentioned above does not sell and a discount of 20% is offered. Calculate the discounted price.

.....

c) Nadine thinks that the discounted price of the camera could be found by 0.95 x $280. Darren thinks it is 0.92 x $280.

Who is right? ..... Show why.

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23 Ratio 3

A Read and Solve

Example : An energy mixture is made with dried fruit, chocolate and nuts in the following weight ratio dried fruit : chocolate : nuts = 7 : 2 : 4.

a) If the mixture contains 80 g of chocolate, what is the weight of the dried fruit and what is the weight of the entire mixture?

b) If you want to make 1 kg of energy mixture, how much chocolate should it contain?

Working :

a) fruit : choc : nuts = 7 : 2 : 4 = x : 80 : y
Since 80 = 2 x 40, then x = 7 x 40 = 280, y = 4 x 40 = 160.

Answer : The mixture contains 280 g of dried fruit and the total mixture weighs 520 g.

b) 7 + 2 + 4 = 13, so $\frac{2}{13}$ of the mixture is chocolate; that is $\frac{2}{13}$ of 1000 grams = 154 grams (to nearest gram).

1 A student of Bay College did a survey on students' hair colours. In a large random sample of students the ratio of hair colours was as follows :

black : brown : blonde : red = 8 : 11 : 5 : 1.

The sample had 45 blonde-haired students in it.

a) How many students in the sample had brown hair?

.....

.....

.....

b) How large was the random sample?

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2 The mixing ratio of normal strength cordial drink is :

cordial : water = 2 : 7.

a) How much water should be added to 120 mL of cordial?

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b) Adam wants to make 2 L of cordial drink. How much cordial should he use?

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B Try These

1 When Anna is up in her room, the ratio of time chatting to friends to time spent doing homework is 2 : 3.

a) If the homework will take Anna 75 minutes, how long can we expect Anna to spend up in her room?

b) If Anna spends 75 minutes in her room, how long is she talking to her friends?

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2 In a bunch of 15 pink and white tulips the ratio pink : white = 2 : 3. One white tulip is removed. What is the new ratio of colours?

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3 Flask A holds 4 times as much water as flask B. Flask C holds $\frac{2}{3}$ of the amount of water in flask A. Write the amount of water in the flasks as a ratio, A : B : C.

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4 Two identical jerry-cans contain mixtures of petrol and oil. Both are full, but in one the ratio petrol to oil is 19 : 1, in the other it is 3 : 1. These jerry-cans are poured into an empty tank. Find the ratio petrol to oil in the tank.

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25 Using a Ratio Table 2

A Conversions

Conversion problems like adjusting proportions in recipes, exchanging money, etc, can be made simple with the use of ratio tables.

Example : To make 20 marshmallow treats, mix 220 g of butter, 36 marshmallows and 5 cups of krispie crumbs.
Write the ingredients for 35 treats.

Working :

treats (number)	butter (g)	marshm. (number)	crumbs (cups)
20	220	36	5
35	x	y	z

?

Since the scale factor is hard to work out, we use the diagonal multiplication rule.

$20 \times x = 35 \times 220 \Rightarrow x = 35 \times 220 \div 20 = 385$ g butter
$20 \times y = 35 \times 36 \Rightarrow y = 35 \times 36 \div 20 = 63$ marshmallows
$20 \times z = 35 \times 5 \Rightarrow z = 35 \times 5 \div 20 = 8\frac{3}{4}$ cups of crumbs

1 Amy wants to make a cake for which she needs to mix 175 g of sugar, 125 g of flour and 50 g butter.

Amy has only 140 g of sugar. How should she adjust the flour and butter?

sugar (g)	flour (g)	butter (g)

.....

.....

.....

.....

2 Suppose one NZ dollar buys 0.62 Euros or 0.79 US dollars.

a) How many NZ dollars do you pay for one thousand Euros?

NZ$	Euro
1	*0.62*
	1000

.....

.....

.....

b) At the airport a souvenir costs US$15.75. How much would that be . . .

NZ$	Euro	US$

i) in NZ dollars? .....

ii) in Euros? .....

B Rates

Example : A car is travelling at a speed of 70 km per hour.
a) How long does it take to go a distance of 40 km?
b) How far does it travel in 25 minutes?

Working :
Set up the ratio table as shown.
Do the calculations.

km	min
70	60
40	a
b	25

e.g. $\frac{a}{60} = \frac{40}{70} \Rightarrow a = \frac{40}{70} \times 60$

$\frac{b}{70} = \frac{25}{60} \Rightarrow b = \frac{25}{60} \times 70$

Answer : a) time is 34 minutes b) distance is 29 km.

1 6 metres of fabric are needed to make covers for 15 cushions.

a) How many metres of fabric are needed to cover 25 cushions?

.....

b) How many cushions can be made with 14 metres of fabric?

.....

m fabric	# cushions

2 On average Harry's car uses 8.6 litres of petrol per 100 km.

a) How many litres does Harry's car use on a 78 km trip?

.....

b) What is the car's fuel consumption rate measured in km per litre?

.....

.....

3 A cup is placed under a leaking tap. The 220 mL cup is filled up with water in 1 hr and 20 min. How many litres of water would leak out in 10 hours?

.....

4 A car travels at a constant speed on the motorway; in 8 minutes it covered 12 km.

a) How long would the car take to travel 22 km?

.....

b) Calculate the car's speed in km/h.

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A Hire Purchase

1

Martin has been saving for an iPod dock. He likes the dock in this advertisement but he does not have enough money yet to pay cash.

a) i) What is the regular price for this wireless speaker?

.....

ii) What percentage discount is the shop offering?

.....

.....

b) If Martin buys the speaker now on hire purchase, what is the total cost of deposit and weekly repayments?

.....

.....

2 A freezer with a cash price of $2700 is bought on hire purchase with a deposit of 20% and 36 monthly instalments of $85.

a) How much is the deposit? .....

.....

b) What amount will be on loan and for how many years?

.....

.....

c) How much extra do you pay for the freezer on hire purchase?

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d) How will the shop justify this difference in price?

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B Repayments

Hire Purchase is a process by which an item becomes the property of the hirer after a number of payments. The hirer pays a certain amount of money as a deposit, then interest is calculated on the outstanding amount at the start of the repayment period. No consideration is given for the fact that the amount outstanding is being reduced during the period of repayments.

Example : A television is for sale at a price of $679.95. Hire purchase is offered with a 10% deposit and monthly repayments over 2 years at 15% interest per annum.

a) Calculate the deposit.
b) Calculate the total amount paid for the TV in 2 years.
c) Calculate the monthly repayments.

Working : a) Deposit = 0.1 x $679.95 = $68
b) Remainder with 2 years interest =
$611.95 + 2 x 0.15 x $611.95 = $795.54
Total paid = $795.54 + $68 = $863.54
c) Monthly repayments = $\frac{\$795.54}{24}$ = $33.15

1 A washing machine is for sale at $1450. On Hire Purchase (HP) a deposit of 15% is required and terms are monthly repayments for 3 years at 12% interest per annum (p.a.).

a) Calculate the deposit .....

.....

b) What is the total amount paid for the washing machine in 3 years.

.....

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c) Calculate the monthly payments.

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.....

2 A car dealer offers a car for $39 995 cash or $2000 deposit with weekly repayments at 14% p.a. for 5 years.

a) What percentage of the price is the deposit?

.....

b) Calculate the weekly repayments. .....

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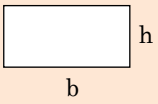
45 Area of Basic Shapes 1

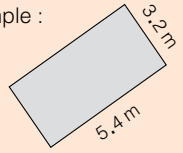
A Formula

The **area** of a shape is the amount of flat surface that the shape covers. This amount of space is measured in cm², m², etc

The formula to calculate the area of a rectangle is $A = b \times h$ where b is the length of the base and h the height.



Example :



The area of this rectangle is

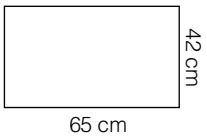
$A = 5.4 \times 3.2$

$= 17.28 \text{ m}^2$

$= 17 \text{ m}^2$ (2 sf)

1 Calculate the area of these rectangles. Remember to write the units.

a)



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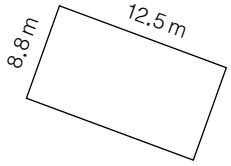
A = ...

b)



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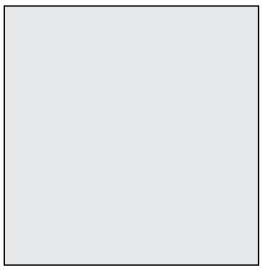
A = ...

2a) Take measurements to calculate the area of this square.



.....

A = ...

b) How many cm² are there in one square metre?

.....

3 The area of a rectangle is found to be 15 cm².
Write possible dimensions (base, height) on these sketches.

A = 15 cm²

A = 15 cm²

4 The area of a rectangle is 8 cm².
Its base is 2.5 cm.
Calculate the height.

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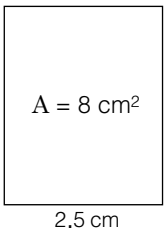
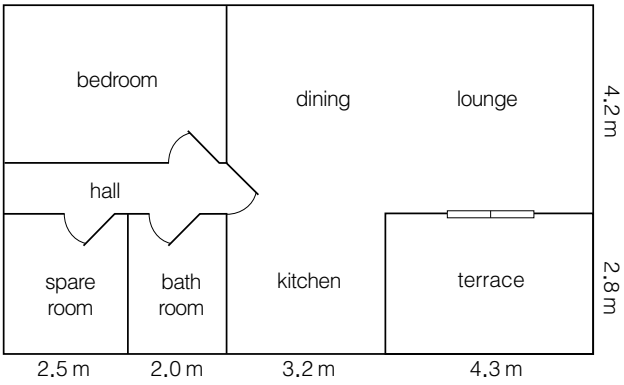


B House and Garden

1



This diagram shows Gran's flat.

a) Work out the area of the bathroom.

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b) The hall is 90 cm wide. Calculate the area of the bedroom.

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c) Calculate the area of the open plan kitchen-dining-lounge.

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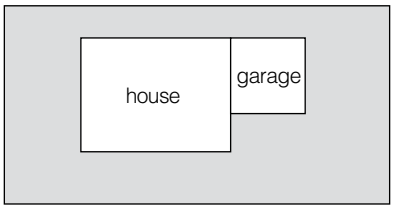
2



scale
1 cm represents 6 m

a) On the plan the garage measures 1 cm by 1 cm.
Its area is 1 cm².

In reality the garage measures ..... by .....

Its area is . .....

b) Take measurements and work out the area of the house.

.....

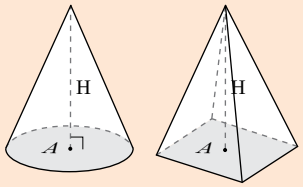
.....

.....

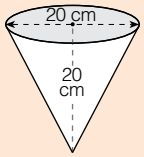
59 Volume of Basic Solids 3

A Changing Cross-sections

Cones and pyramids are solids with an ever changing cross-section, they end in a point called the apex.
Their volumes are found with the formula
$V = \frac{1}{3} \times A \times H$,
where A is the area of the base and H the perpendicular height from the base to the apex.



Example :
How many litres of water could this cone hold?



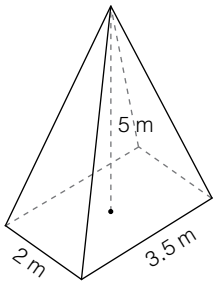
Working : $r = 10$; $A = \pi \times 10^2$;
$H = 20$.
$V = \frac{1}{3} \times \pi \times 10^2 \times 20$

key in : [1] [÷] [3] [x] [π] [x] [10] [x^2] [x] [20] [EXE]

Answer : $V = 2094\ cm^3 = 2.1$ litres (2 sf)

1 Calculate the volumes of these solids.

a)



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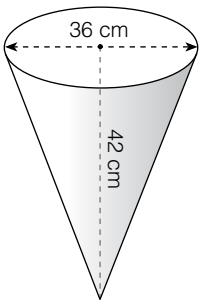
b)



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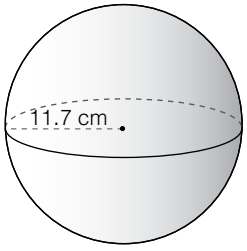
The volume of a sphere is found with the formula $V = \frac{4}{3}\pi r^3$.

2



A basketball has a radius of 11.7 cm. Calculate the volume of the basketball.

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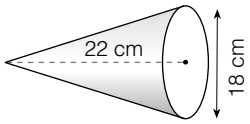
B More Practice

1 Calculate the volume of these solids.

a)



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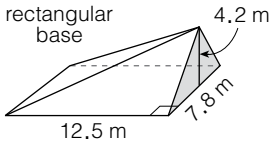
b)



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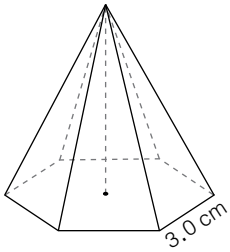

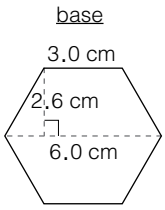

This pyramid is 7.5 cm high, it has a regular hexagon as its base. Calculate the volume of the pyramid.

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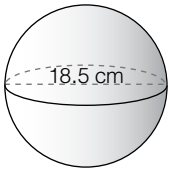
3 Calculate the volume of these solids.

a)



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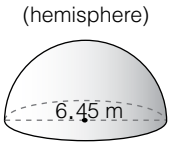
b)



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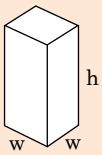
63 Calculating Dimensions 2

A Formulas and Your Graphic Calculator

Your graphic calculator is a handy tool when working with formulas.

Example : The volume (V cm^3) of a cuboid with a square base w cm wide and a height h cm is calculated with the formula : $V = w^2h$.



a) Calculate h, if $V = 82$ cm^3 and $w = 6.9$ cm.

b) Calculate w, if $V = 127$ cm^3 and $h = 8.5$ cm.

Working : (steps for the **Casio fx 9750** or **9860 GIII**)

i) Select EQUA from the main menu, then select F3:Solver. Delete any existing equation with DEL(F2), Yes(F1).

ii) Now use the ALPHA keys to enter the formula :

[ALPHA] [V] [=] [ALPHA] [W] [x^2] [ALPHA] [H]

(note: the = sign is yellow, entered as [SHIFT] [•]).

Press [EXE].

a) We know V and W and wish to calculate H, so,

when [V=] is highlighted you enter [82] [EXE];

when [W=] is highlighted you enter [6.9] [EXE].

When [H=] is highlighted you select SOLV (F6).

Answer : height = 1.7 cm (2 sf).

b) Select REPT (F1). This time we know V and H. So use the REPLAY arrows to move the highlight strip up and down, and change V to 127 and H to 8.5.
Then highlight W and select SOLV (F6).

Answer : width = 3.9 cm (2 sf).

1 Enter the formula for volume of a cylinder : $V = \pi r^2 h$.

a) Calculate the height of a cylindrical milk container, which has a radius of 1.4 m and a volume of 25 m^3.

.....

b) The volume of a small tin of coconut milk is 100 mL and its height is 6.5 cm. Calculate the radius of the can.

.....

2 The formula for the surface area of a cone is :
$A = \pi r^2 + \pi rs$, where r is the radius of the bottom circle and s the slant height.

a) Calculate the surface area of a cone with radius is 8.2 cm and the slant height is 25 cm.

.....

b) Calculate the length of the radius, if the surface area is 28 cm^2 and $s = 3.7$ cm.

.....

c) Calculate the slant height, if the largest diameter of the cone is 5.4 m and the surface area is 56 m^2.

.....

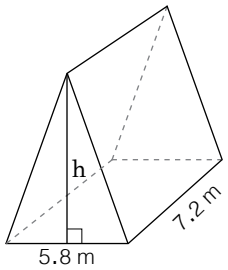
.....

B Practise Your Calculator Skills

If you are using a graphic calculator to solve problems, you must show how you did it.

For each of the following questions write down just one formula you are going to use to solve the problem and the values of the known variables. Then solve and round sensibly.

1 This triangular prism has a volume of 158 m^3.
Calculate the height of the triangular face.



Working : I use the formula

.....

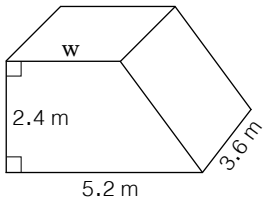
.....

Known variables are : .....

.....

The height is .....

2 The volume of an attic is 32 m^3. The floor measures 5.2 m by 3.6 m and the ceiling is 2.4 m high. Calculate the width of the ceiling.



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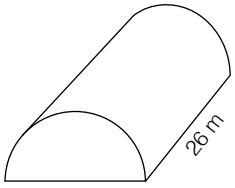
3 A plastic tunnel house has the shape of a half cylinder and is 26 metres long. The area of the roof is 335 m^2.
Calculate the area of soil covered by the tunnel house.
Show your working.



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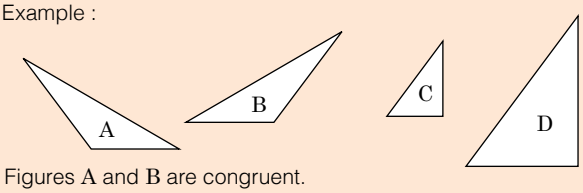
A Congruent or Similar?

Congruent figures have the same shape and the same size. By rotating them or flipping them over you can fit congruent shapes exactly on top of each other.

Similar figures have the same shape but not the same size. The sides of the smaller figure are multiplied by a **factor** (k) to get the sides of the larger shape.

Example :

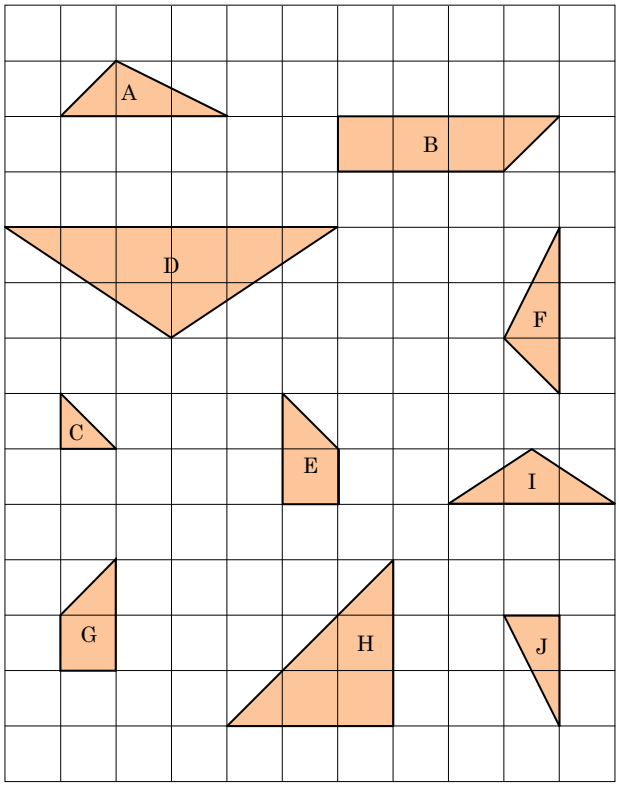


Figures A and B are congruent.
Figures C and D are similar. The scale factor for the sides is 2 (k = 2).

1a) Name pairs of congruent shapes in the diagram below.

A and ……; ……

……



b) Name pairs of similar shapes.
For each pair, give the scale factor k.

…….. and ……., k = ……; ……;

……

……

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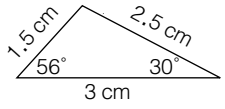
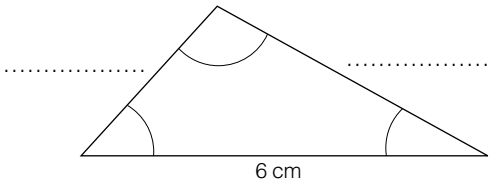
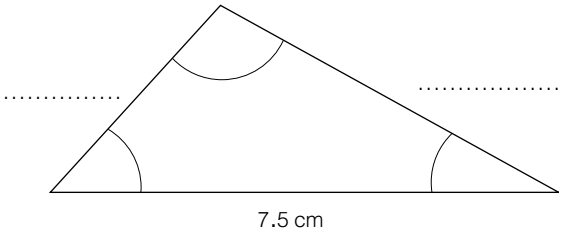
B Angles and Sides

Remember : Angles inside a triangle add to 180°

1a) These three triangles are similar (they are not drawn to scale). Work out all sides and all angles of the triangles.

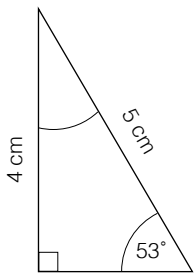






B

b)



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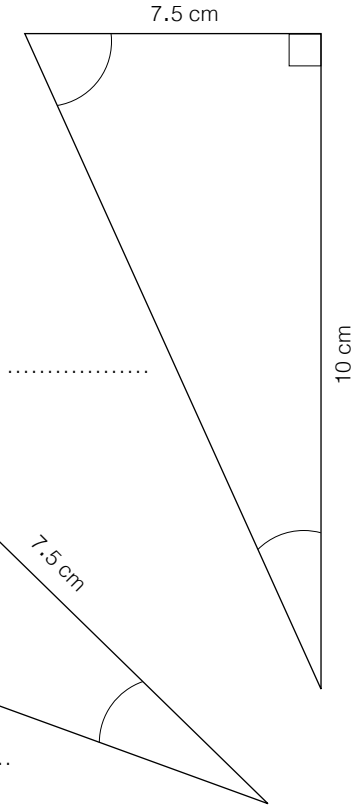
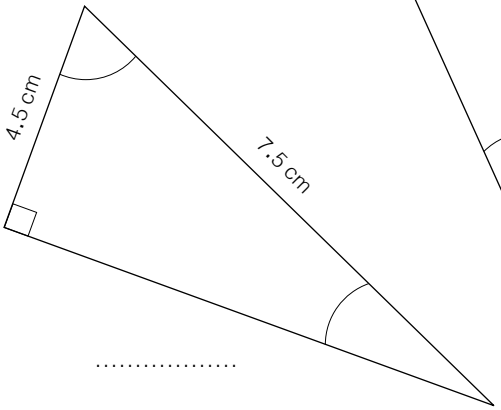
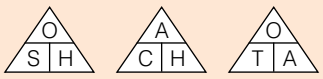




Basics of Trigonometry 1

A Four Steps to Calculate a Side

If in a right-angled triangle you know the size of one more angle and the length of one side, then you can use the ratio triangles to calculate any of the other two sides.

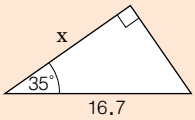


The calculation has 4 steps :

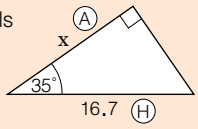
1) In the right-angled triangle label two of the sides with **H** (hypotenuse) or **O** (opposite) or **A** (adjacent).
Only label the side you know and the side you want to know.
2) Choose the relevant ratio triangle : SOH, CAH, or TOA.
3) Substitute known values into the ratio triangle.
4) Calculate the length of the side using your calculator and round sensibly.

Example : Calculate **x**.

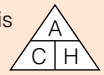
Working :



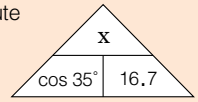
1) Labels



2) With labels A and H the choice is

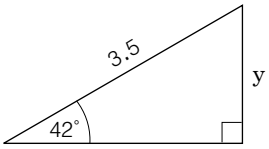


3) Substitute values.



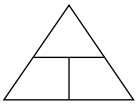
4) Calculate and round.
$x = \cos 35^\circ \times 16.7$
$= 13.7$ (3 sf)

1 We will use the 4 step method to calculate the length of side **y**.

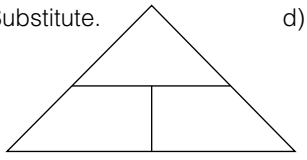


a) Label sides '**y**' and '3.5'.

b) Choose SOH, CAH or TOA.



c) Substitute.



d) Calculate and round **y**.

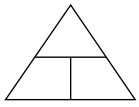
y = ...

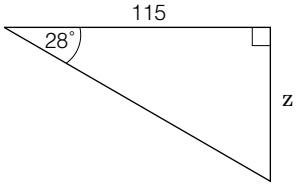
.....

2 Calculate the length of side **z** in four steps.

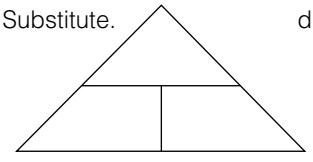
a) Label sides '**z**' and '115'.

b) Choose SOH, CAH or TOA.





c) Substitute.



d) Calculate and round **z**.

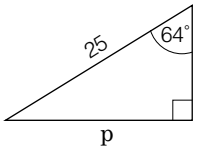
z = ...

.....

B On Your Own

1 Calculate the labelled sides, round sensibly.

a)



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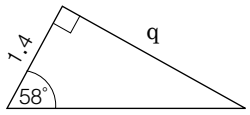
b)



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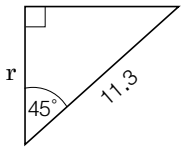
c)



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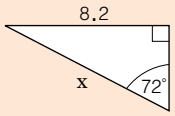
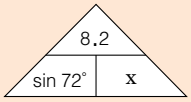
Example : Calculate side **x**.

Working : **x** has label H
8.2 has label O

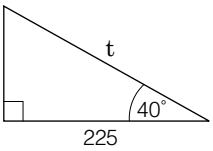




$$x = \frac{8.2}{\sin 72^\circ} = 8.6 \text{ (2 sf)}$$

2 Calculate the labelled sides, round sensibly.

a)



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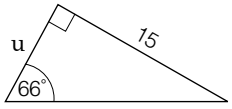
b)



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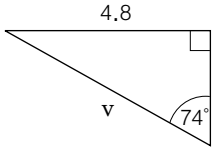
c)



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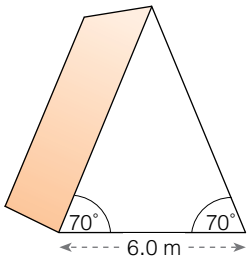
A Draw Extra Lines

Hint : If there are no right-angled triangles in the diagram, you need to draw some extra lines to create them.

1 

Duncan drew a sketch of his symmetrical A-frame house. He needs to replace the sheets of iron on the roof, which run all the way down to the ground.

What length do the sheets of iron need to be?

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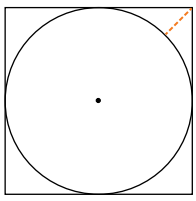
2 A circle with radius 8.0 cm fits snugly inside a square. Calculate the distance from a corner of the square to the circumference of the circle.



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3 A rhombus with sides of 53 mm has a diagonal of 56 mm. Calculate the length of the other diagonal.

……

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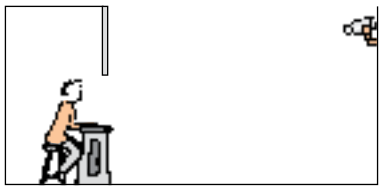
B Video Surveillance

1 A bank installed a video camera on a wall at a height of 2.5 m. It is aimed at the head of the bank teller who sits at a distance of 5.7 m from the wall. The head of the teller is 1.6 m above the ground.



Calculate the angle of depression the camera should be set at.

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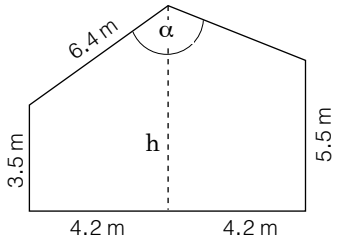
2 This is a cross section of a house. The ridge of the roof is exactly halfway the width of the house.



a) Calculate the height of the house (h).

……

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……

……

b) Calculate the angle between the two roof sections (α).

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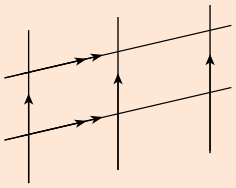
A Parallel Lines

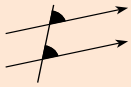
If in a diagram a set of lines is marked with the same type of arrows then we know that these lines are **parallel**.



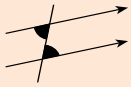
This diagram shows 3 parallel lines going up. They are crossed by another pair of parallel lines.

A line crossing parallel lines is called a **transversal**. There are 3 rules about angles formed by a transversal and parallel lines.

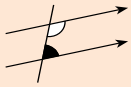
Rule : Corresponding angles on parallel lines are equal. (corr ∠s // lines are =)



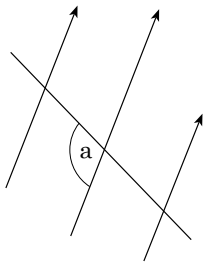
Rule : Alternating angles on parallel lines are equal. (alt ∠s // lines are =)



Rule : Co-interior angles on parallel lines add to 180°. (co-int ∠s // lines add to 180°)



1

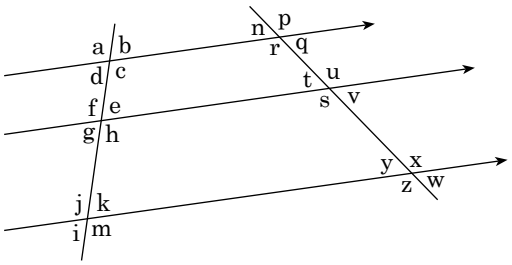


a) Colour red the two angles that are corresponding to angle a.

b) Colour blue the angle that is alternating with angle a.

c) Colour green the angle that is co-interior with angle a.

2



a) Why is c = h? .....

.....

b) Why is f + g = 180°? .....

.....

c) Why is t = q? .....

.....

d) Why is s + y = 180°? .....

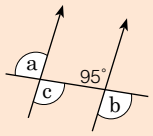
.....

e) Which of these is true? Circle the correct one.

A c + r = 180° B c + t = 180° C c + k = 180°

B Giving Reasons

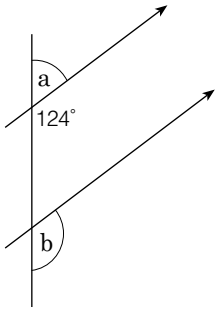
Example :



Calculate angles a, b and c.
Give reasons for your answers.

Answer : a = 95° (corr ∠s // lines are =)
b = 95° (vert opp ∠s are =)
c = 95° (alt ∠s // lines are =)

1



Calculate angles a and b.
Give reasons for your answer.

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2 Calculate angles c and d.
Give reasons for your answer.

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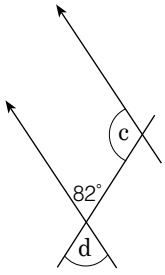
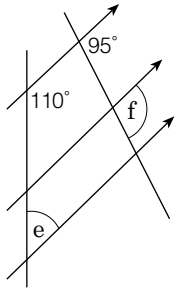


3



Calculate angles e and f.
Give reasons for your answer.

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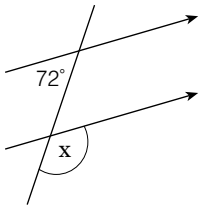
4



Calculate angle x.

There is no rule connecting x with the marked angle of 72°.

In the diagram mark another angle which could be used as a stepping stone. Label this angle with w and work out its size.

w = ... reason .....

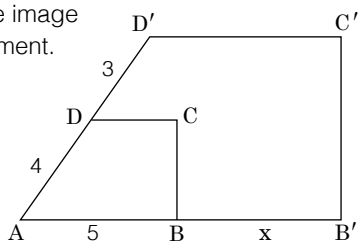
.....

x = ... reason .....

.....

A An Enlargement

1 Trapezium $AB'C'D'$ is the image of $ABCD$ after an enlargement.
$\overline{AB}$ = 5 cm,
$\overline{AD}$ = 4 cm,
$\overline{DD'}$ = 3 cm.

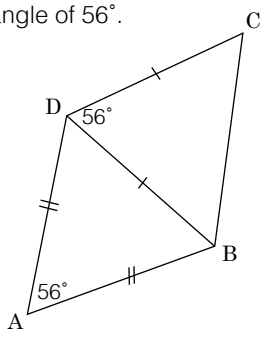


a) Calculate the length of $\overline{BB'}$, marked x.

b) Also given : $\angle BAD = 68^\circ$, $\angle ABC = 90^\circ$.
Calculate the areas of the two trapezia.
Explain your reasoning.

B Two Triangles

1 Two isosceles triangles have one side in common as shown in the diagram which is only a sketch.
Both triangles have a top angle of 56°.



a) Are the two triangles congruent? Justify your answer.

b) What is the best name for quadrilateral $ABCD$?
Justify your answer.

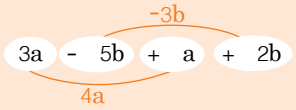
Simplifying Expressions 4

A In Your Head

Instead of reorganising the terms, collecting like items can be done in your head.

Example : Simplify $3a - 5b + a + 2b$

Working in your head:



Answer : $3a - 5b + a + 2b = 4a - 3b$

1 Simplify.

a) $5a + 2b - a + 2b$ .....

b) $4y + z - 4y + z$ .....

c) $3a + 4b - 6a + b$ .....

d) $y^2 - 2 + 3y^2 - 8$ .....

e) $-2a + 4b + 3a - 6b$ .....

f) $4x^2 - 5x + 3x + 2$ .....

g) $6ab - 2b + 3b + 4$ .....

Like terms are terms with the same combination of **variables** and **exponents** e.g. $2a^2b$ is like $3a^2b$, but not like ab^2.

Example : Simplify $3xy^2 + 2x^2y - xy^2$

Working : Since $3xy^2 - xy^2 = 2xy^2$, then

$3xy^2 + 2x^2y - xy^2 = 2xy^2 + 2x^2y$

2 Simplify (if possible).

a) $3ab + 2ba$ .....

b) $3a^2b + 2a^2b$ .....

c) $3a^2b + 2ab^2$ .....

3 Simplify if possible.

a) $x - 3xy + 5xy$ .....

b) $5ab - 2ba + b^2$ .....

c) $2k^2m + 3km^2$ .....

d) $a^2b + ab^2 - 3a^2b$ .....

B A Mixed Bag

When simplifying expressions, always check what type you are dealing with.

- multiplication ? multiply numbers and use index notation.
- division / fraction ? use cancellation.
- adding / subtracting ? collect like terms.

1 Simplify.

a) $\frac{15x^3}{20x}$ .....

b) $4a - 6 + 6 + a$ .....

c) $2y^3 \times 3y^2$ .....

d) $(-3x^2)^2$ .....

e) $3y^2 - 5y + y$ .....

f) $\frac{(2a)^2}{4a}$ .....

g) $a^2 + a^2 + a^2$ .....

2 Simplify (if possible).

a) $2ab + b^2 - 3ba$ .....

b) $2ab \times b^2 \times -3ba$ .....

3 Remember the order of operations when you simplify these.

a) $\frac{4ac \times 9a^2}{6c^2}$ .....

b) $\frac{4a^2 + 2a^2}{3a}$ .....

c) $\frac{y + y + y}{3}$ .....

d) $\frac{a^2 \times a^2 \times a^2}{a^2}$ .....

e) $\frac{3ab - ba}{b^2}$ .....

Solving Linear Equations 2

A Shorthand Notation

Drawing a flow-chart for each equation takes a lot of space. Usually we imagine the diagram and write down, in shorthand, the steps that show how we unwrap the variable.

Examples : $3y + 5 = 11$ $(- 5)$
$3y = 6$ $(\div 3)$
$y = 2$
check : $3 \times 2 + 5 = 11$

$\frac{y + 5}{3} = 9$ $(\times 3)$
$y + 5 = 27$ $(- 5)$
$y = 22$
check : $\frac{22 + 5}{3} = 9$

$\frac{y}{5} + 3 = 2$ $(- 3)$
$\frac{y}{5} = -1$ $(\times 5)$
$y = -5$
check : $\frac{-5}{5} + 3 = 2$

$\frac{2y}{3} + 1 = 5$ $(- 1)$
$\frac{2y}{3} = 4$ $(\times 3)$
$2y = 12$ $(\div 2)$
$y = 6$
check : $\frac{2 \times 6}{3} + 1 = 5$

1 You could write the flow-chart on scrap paper. Write down in the brackets the steps needed to unwrap the variable. Check your answers.

a) $2y + 6 = 11$ (.....)
$2y =$ ..... (.....)
$y =$ .....
check : .....

b) $\frac{y - 3}{2} = 6$ (.....)
..... = ... (.....)
..... = ...
check : .....

c) $\frac{y}{3} + 8 = 10$ (.....)
..... = ... (.....)
..... = ...
check : .....

d) $3(y - 4) = -9$ (.....)
.....
.....
check : .....

e) $\frac{4y}{5} = -6$ (.....)
.....
.....
check : .....

f) $5(\frac{y}{2} - 1) = 15$ (.....)
.....
.....
.....
check : .....

B Solve

1 Solve these equations. Show the steps.

a) $\frac{x - 7}{3} = 2$
.....
.....

b) $10y - 1 = 13$
.....
.....

c) $3(x - 2) = -9$
.....
.....

d) $2w - 4 = -1$
.....
.....

e) $\frac{a}{3} - 1.6 = 0.5$
.....
.....

f) $-3x + 10 = -2$
.....
.....

g) $\frac{3x + 1}{2} = 5$
.....
.....
.....

h) $3(2a + 5) = 18$
.....
.....
.....

i) $\frac{2(y - 1.5)}{3} = -4$
.....
.....
.....

Factorising Quadratics 3

A Complications

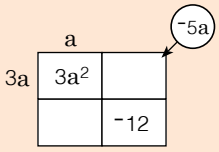
Factorising a trinomial becomes more complicated if there is a number in front of a^2. This number is the **coefficient** of a^2.

Example : Factorise $3a^2 - 5a - 12$

Working : $3a^2$ is made by $3a \times a$

-12 can be made by . . .

-1×12	1×-12
-2×6	2×-6
-3×4	3×-4



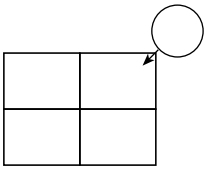
Check that -3×4 works, but only if 4 goes with $3a$ and -3 goes with a.
(Not the other way round.)

Answer : $3a^2 - 5a - 12 = (3a + 4)(a - 3)$

1 Factorise.

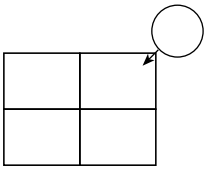
a) $2y^2 + 7y + 3$

= (.....) (.....)



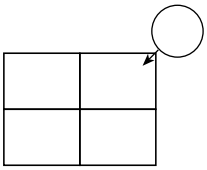
b) $3x^2 + 4x - 4$

= (.....) (.....)



c) $5a^2 - 13a - 6$

.....



B Tricky Stuff

If the **coefficient** of a^2 is not a prime, factorising can become a real puzzle. Ask your teacher for handy tips.

1 Factorise these.

a) $6x^2 - x - 2$

.....

b) $6a^2 - 25a + 4$

.....

c) $4y^2 + 8y + 3$

.....

d) $4n^2 + 4n - 15$

.....

e) $4a^2 + 5a - 6$

.....

f) $8x^2 - 29x - 12$

.....

working space

C More Practice

1 Factorise

a) $3a^2 - 10a - 8$ .....

b) $9t^2 - 1$ .....

c) $20y^2 - 12y$ .....

d) $5x^2 - x - 6$ .....

e) $6p^2 - 7p - 3$ .....

f) $3n^2 - 5n - 12$ .....

g) $25a^2 - 16$ .....

h) $9x^2 + 24x + 16$ .....

i) $12x^2 - 31x - 30$ .....

Solving Quadratic Equations 3

A Zero on the Side

A **quadratic equation** can only be solved by **factorisation** if one side of the equation equals zero.

Example : Solve $x^2 + 5x = 3x + 3$

Working :
$x^2 + 5x - 3x - 3 = 0$ (subtract $3x + 3$)
$x^2 + 2x - 3 = 0$ (simplify)
$(x + 3)(x - 1) = 0$ (factorise)

Solution : $x = -3$ or $x = 1$

1 Solve.

a) $x^2 - 3x + 4 = 14$

b) $n^2 - 10n + 21 = 12$

c) $a^2 - 7a - 8 = a + 12$

d) $2y^2 + 5y - 10 = 4y + 5$

B But Wait, There's More . . .

1 Solve.

a) $3y^2 - 7y + 5 = 2y - 1$

b) $(2k + 1)(k + 1) = 6$

c) $(2x + 1)^2 = (x - 4)^2$

d) $(p - 2)(p + 6) = (3p + 4)(p - 3)$

A Plotting a Graph

With an equation we can make a table, and with a table we can make a graph. If the x-value can be any (decimal) number, then the graph is a continuous line.

1 Make a table for each rule then plot the points and connect them. There will be 2 graphs on each grid.

a) $y = -x + 2$

x		y
-1		
0		
1		
2		

b) $y = \frac{x - 3}{2}$

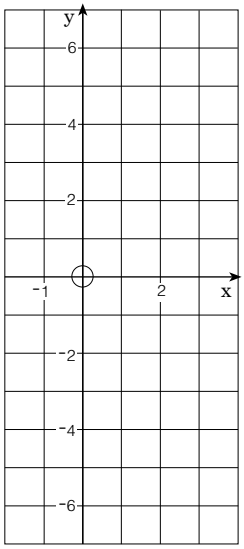
x		y
-1		
0		
1		
2		

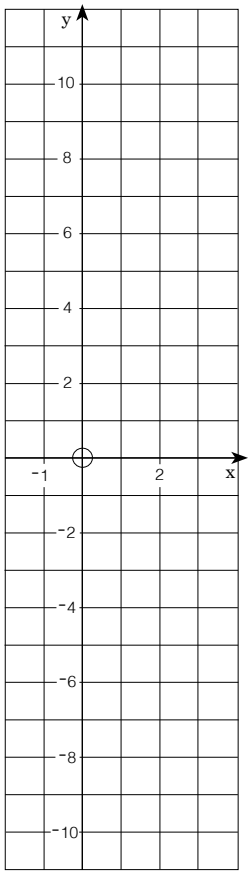
c) $y = 5 - 3x$

x		y

d) $y = 4(x - 1)$

x		y





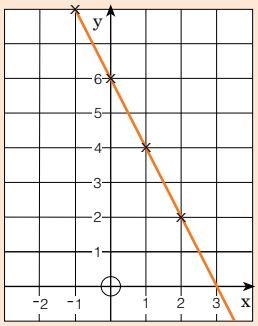
B Rules in the Form ax + by = c

Example : a) Make a table for the rule $2x + y = 6$
b) Plot the graph.

Working : a) Substitute x, into the rule, calculate y.

x	2x + y = 6	y	(x, y)
-1	-2 + y = 6	8	(-1, 8)
0	0 + y = 6	6	(0, 6)
1	2 + y = 6	4	(1, 4)
2	4 + y = 6	2	(2, 2)

b) Plot the points and draw the line.



1 Make a table for each rule then draw the graph. There will be 2 graphs on one grid.

a) $3x - y = 1$

x		y
-1		
0		
1		
2		

b) $x + 2y = 8$

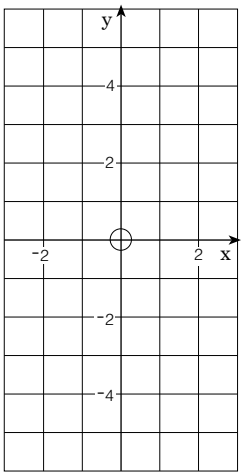
x		y
-1		
0		
1		
2		

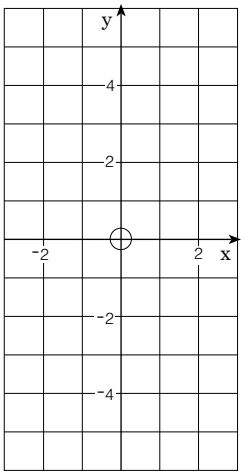
c) $2x + \frac{1}{2}y = 1$

x		y
-1		
0		
1		
2		

d) $3x - 2y = 0$

x		y
-1		
0		
1		
2		





A Rearrange

If the rule is not written in the form $y = mx + c$ we can rearrange it.

Examples : Find gradient and y-intercept for these lines.

a) $y = \frac{2x - 1}{3}$ b) $3x + 4y = 6$

Working : a) $y = \frac{2x}{3} - \frac{1}{3}$

$y = \frac{2}{3}x - \frac{1}{3}$

$m = \frac{2}{3}$, $c = \frac{-1}{3}$

b) $4y = -3x + 6$

$y = \frac{-3x}{4} + \frac{6}{4}$

$y = \frac{-3}{4}x + 1\frac{1}{2}$

$m = \frac{-3}{4}$, $c = 1\frac{1}{2}$

1 Rearrange to find gradient (m) and y-intercept (c).

a) $y = \frac{x + 3}{4}$

.....

.....

m = ..., c = ...

b) $y = \frac{3x + 2}{5}$

.....

.....

m = ..., c = ...

c) $2x + 5y = 5$

.....

.....

.....

m = ..., c = ...

d) $x - 2y = 8$

.....

.....

.....

m = ..., c = ...

e) $\frac{1}{2}x + 3y = 6$

.....

.....

.....

m = ..., c = ...

B Rearrange, then Draw

1 The rule of a line is $3x - 5y = 6$.

a) Work out the intercepts with axes and the gradient.

b) Use the gradient and one intercept to draw the graph.

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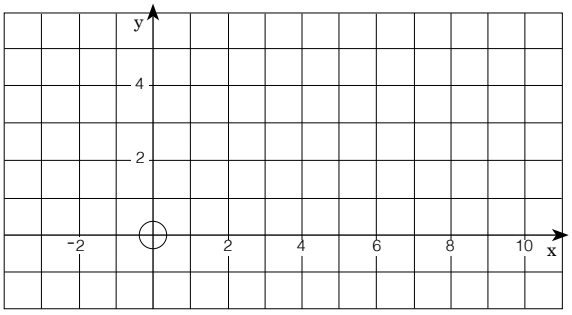

2 The relationship between C and t is given by $15C = 700 - 20t$.

a) Work out the intercepts with axes and the gradient.

b) Use the gradient and one intercept to draw the graph.

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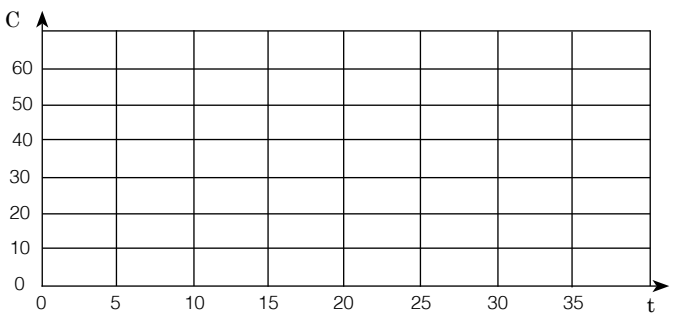


Factorised Equations 3

A Equation $y = \pm(x - p)(x - q)$

The parabolas in this exercise are not drawn to scale, but it is known their equation is of the form $y = \pm(x - p)(x - q)$. For each parabola you are asked to . . .

a) Write the equation,
b) Work out the y-intercept,
c) Work out the coordinates of the vertex.

1a) Equation :

.....

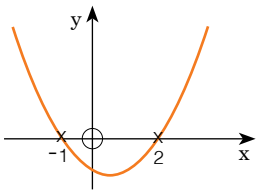
b) y-intercept :

.....

c) vertex : .....

.....

.....



2a) Equation :

.....

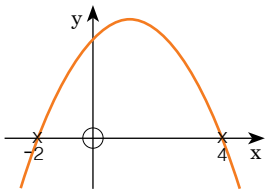
b) y-intercept :

.....

c) vertex : .....

.....

.....



3a) Equation :

.....

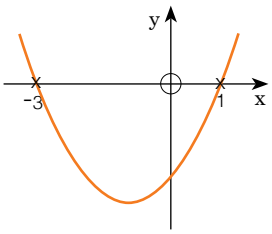
b) y-intercept :

.....

c) vertex : .....

.....

.....



4a) Equation :

.....

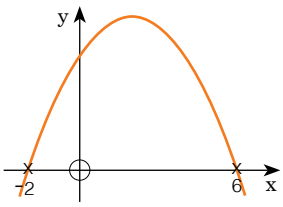
b) y-intercept :

.....

c) vertex : .....

.....

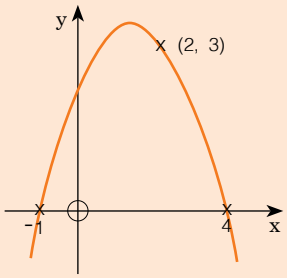
.....



B Equation $y = a(x - p)(x - q)$

Example :
Write an equation for this parabola.

Working : Using the x-intercepts
$y = a(x + 1)(x - 4)$
The value of a can be calculated as follows :
since (2, 3) is on the parabola, we substitute $x = 2$ and $y = 3$ in the equation :
$y = a(x + 1)(x - 4)$
$3 = a(2 + 1)(2 - 4)$
$3 = a \times 3 \times -2$ then $3 = -6a$ so $a = -\frac{1}{2}$.
Equation : $y = -\frac{1}{2}(x + 1)(x - 4)$



1 Write an equation for each graph.

a) .....

.....

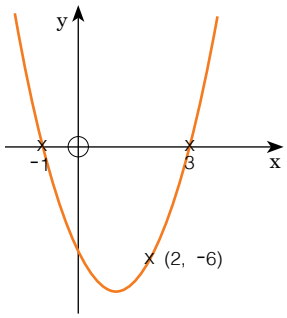
.....

.....

.....

.....

.....



b) .....

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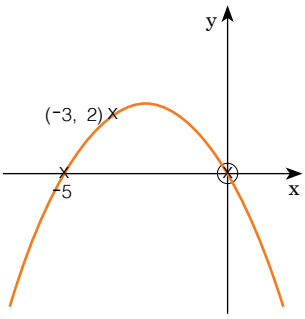

c) .....

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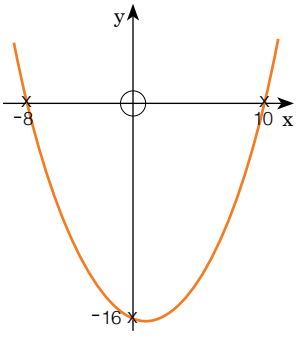

A Practice Questions 1.4

1 A child's toy rocket is fired up into the sky, it reaches 98m then starts to fall and lands 14 metres away. A graph of the rocket's journey is shown below.

How far does it travel horizontally when it is above 40m high.

……

……

……

……

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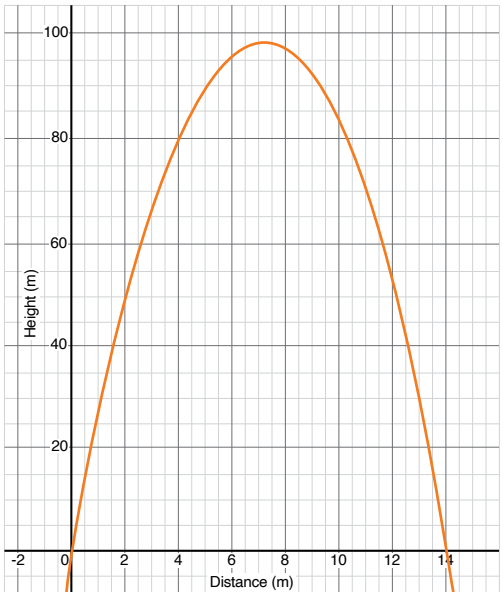
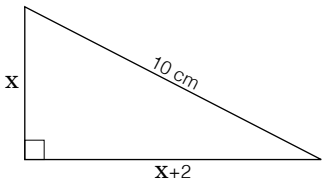


2a) A right-angled triangle has a hypotenuse of length 6 cm and one shorter side of length x cm. The other side can be expressed as $x+4$ cm.

Write down an equation involving x representing the Pythagorean theorem for this triangle.

……

……



b) Solve the equation from part (a) in order to calculate the lengths of the two missing sides of the triangle.

……

……

……

……

3 Show that $\frac{4}{x} + \frac{x+1}{3} = \frac{x^2+x+12}{3x}$

……

……

……

……

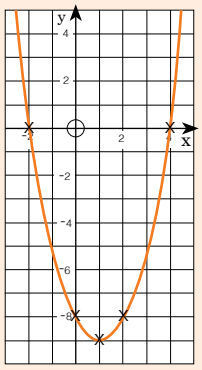
Working Area

189 Answers

Pages 161 - 176

Page 161 - Factorised Equations 1

A1 a) $y = 2x - 4 = -8$ b) $x = -2$ or $x = 4$

c) 

d) for vertex $x = 1$, $y = 3 \times -3 = -9$

B1 y-int : (0, 8) ; x-int (−2, 0) (−4, 0) ; vertex : (−3, −1)

B2 y-int : (0, −6) ; x-int (2, 0) (−3, 0) ; vertex : $(-\frac{1}{2}, -6\frac{1}{4})$

B3 a) x-int (−3, 0) (5, 0) ; vertex : (1, −16)

b) x-int (0, 0) (−3, 0) ; vertex : $(-1\frac{1}{2}, -2\frac{1}{4})$

Page 162 - Factorised Equations 2

A1 a) y-int : $(0, -1\frac{1}{2})$; x-int (−3, 0) (1, 0) ; vertex : (−1, −2)

b) y-int : (0, −4) ; x-int (1, 0) (2, 0) ; vertex : $(1\frac{1}{2}, \frac{1}{2})$

c) y-int : (0, −12) ; x-int (2, 0) (−2, 0) ; vertex : (0, −12)

B1 a) $y = (x - 3)(x - 2)$

b) y-int (0, 6); x-int (3, 0)(2, 0) vertex : $(2\frac{1}{2}, -\frac{1}{4})$

B2 a) $y = (x + 1)(x - 3)$
y-int (0, −3); x-int (−1, 0)(3, 0)
vertex : (1, −4)

b) $y = x(x - 2)$
y-int (0, 0); x-int (0, 0)(2, 0)
vertex : (1, −1)

c) $y = (2x + 1)(x - 3)$
y-int (0, −3); x-int $(-\frac{1}{2}, 0)(3, 0)$
vertex : $(1\frac{1}{4}, -6\frac{1}{8})$

Page 163 - Factorised Equations 3

A1 a) $y = (x + 1)(x - 2)$ b) (0, −2)
c) vertex : $(\frac{1}{2}, -2\frac{1}{4})$

A2 a) $y = -(x + 2)(x - 4)$ b) (0, 8)
c) vertex : (1, 9)

A3 a) $y = (x + 3)(x - 1)$ b) (0, −3)
c) vertex : (−1, −4)

A4 a) $y = -(x + 2)(x - 6)$ b) (0, 12)
c) vertex : (2, 16)

B1 a) $y = a(x + 1)(x - 3)$; $-6 = a \times 3 \times -1$ then $a = 2$
equation : $y = 2(x + 1)(x - 3)$

b) equation : $y = -\frac{1}{3}x(x + 5)$

c) equation : $y = \frac{1}{5}(x + 8)(x - 10)$

Page 164 - Writing Quadratic Equations 1

A1 a) $y = \frac{1}{2}x^2 + 1$ b) $y = \frac{1}{3}(x + 2)(x - 3)$

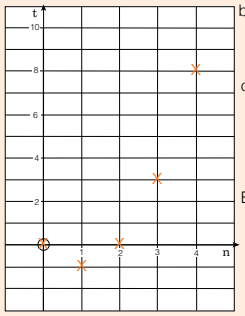
c) $y = -(x + 2)^2 + 3$ d) $y = 0.3\,x\,(x + 3)$

e) $y = -\frac{2}{3}x^2 + 10$ f) $y = 0.08\,(x - 5)^2$

Page 165 - Writing Quadratic Equations 2

A1 a) $y = 2(x + 3)(x + 1)$ $y = 2(x + 2)^2 - 2$

b) $2(x + 3)(x + 1) = 2(x^2 + 4x + 3) = 2x^2 + 8x + 6$
$2(x + 2)^2 - 2 = 2(x^2 + 4x + 4) - 2 = 2x^2 + 8x + 6$

A2 a) 

b) $t = n(n - 2)$;
$t = (n - 1)^2 - 1$
$t = n^2 - 2n$

c) $t = 360$

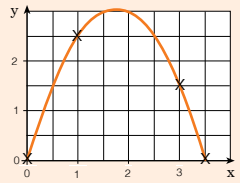
B1 A - k B - g
C - c D - i
E - f F - d
G - b H - l
I - e J - j
K - a L - h

Page 166 - Quadratic Problems

A1 a) $H = 2.2 \times 0.8 = 1.76$ metres

b) 2.2 metres

c) vertex : $d = 0.7$, $H = 2.25$ m ; max height 2.25 m

A2 a) 

b) 3.0625 m

A2 c) Driving through the centre the width of the trailer goes from $x = 0.75$ to $x = 2.75$; the height of the arch is at least 2.0625 metres. So the tractor will fit.

B1 a) 2 m b) Towers are at $x = 50$ and $x = -50$ height = $0.005 \times 50^2 + 22 = 34.5$ m

B2 Origin on the water in the centre : $y = \frac{1}{25}x^2 + 3$ or origin at the base on left pole : $y = \frac{1}{25}(x - 5)^2 + 3$

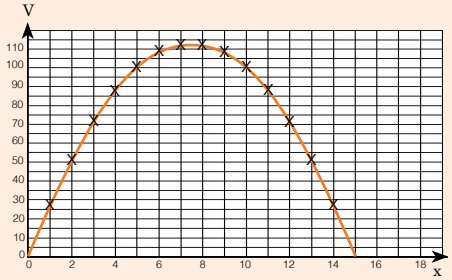
Page 167 - Optimisation Problem

A1 If $x = 3$, then $h = 12$ and $V = 72$ cm^3

A2

x	1	2	3	4	5	6	7	8	9	...
h	14	13	12	11	10	9	8	7	6	...
V	28	52	72	88	100	108	112	112	108	...

For whole number values of x, the maximum volume is reached for $x = 7$ or $x = 8$, $V = 112$ m^3.
The relationship is quadratic because the second difference is −4. The equation for the parabola is $y = ax(x - 15)$ and $a = -2$.
So equation : $y = -2x(x - 15)$. Maximum volume reached when $x = 7.5$ cm, $V = 112.5$ cm^3.

A2 

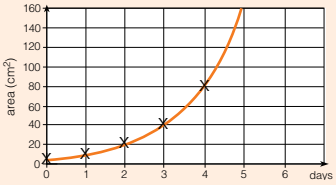
Page 168 - Exponential Patterns 1

A1 a) day 4, area = 80 cm^2 ; day 5, area = 160 cm^2

b) *'On day 3 the mould covered an area of 40 cm^2.'*

c) 5×2^n d) The variable n is in the exponent.

e) When $n = 15$, Area = 5×2^{15} ; Area = 163 840 m^2.

f) 

g) day (−1), area = 2.5 ; day (−2), area = 1.25 or $5 \times 2^{-2} = 1.25$

B1 a) $n = 10$, $t = 59\,049$; rule : $t = 3^n$

b) $n = 10$, $t = 19\,531\,250$; rule : $t = 2 \times 5^n$

c) $n = 10$, $t = 16\,384$; rule : $t = 16 \times 2^n$

B2 Since $16 = 2^4$, the rule becomes $t = 2^4 \times 2^n = 2^{n+4}$

B3 table : x column (top to b) = 1, 2, 3.
y column (top to b) = 20, 80, 320.
work backwards : when $x = 0$, $y = 5$; $y = 5 \times 4^n$.

Page 169 - Exponential Patterns 2

A1 a) $1.35 \times \$92 = \$124.20$

b) $1.08 \times 3000 = 3240$ people

c) $0.30 \times \$22\,000 = \$6600$ d) 0.82×640 L $= 524.8$ L

A2 2013 - $459 000 ; 2014 - $468 180 ;
2015 - $477 543.6

B1 a) Year 2 : $1.10 \times 5500 = \$6050$
Year 3 : $1.10 \times 6050 = \$6655$

b) Amounts are multiplied by the same number : 1.10

c) $A = 5000 \times 1.10^n$

d) (4) 7320.50 ; (5) 8052.55 ; (6) 8857.81 ;
(7) 9743.59 ; (8) 10717.94. Answer : 8 years

C1 a) Year 2012 : population 720, 2013 : population 864
2014 : population 1036

b) $p = 500 \times 1.2^t$

c) In 2025, $t = 15$, $p = 500 \times 1.2^{15}$
population : 7703 penguins.

Page 170 - Writing Equations Using Technology

A1 a) $y = 1.5x^2 + 1$ b) 151

A2 a) $y = 2.25x + 10.35$ b) 32.85

A3 a) $y = 2.5 \times 2.2^x$ b) 6640.0 (1 dp)

A4 a) $y = 0.5x^2 + 35x + 3$ b) 403

A5 a) $y = 1000 \times 1.1^x$ b) 2593.7 (1 dp)

Pages 171 - 173 - Practice Investigation 1.2

A Total Volume - 758 cm^3
Weight = 0.91 kg Resin cost
Exclusive GST + $16.62
Stick cost Exclusive GST = $57.60
Total Cost = $74.22

B 12.73 cm C 14.38 cm

Pages 174 - 176 - Practice Paper 1.4

A1 $y = -2x^2 + 28x$
Points of intersection (1.169,30) and (12.831,30)
Horizontal distance travelled = 11.662m

2 a) $x^2 + (x + 2)^2 = 100$

b) $(x + 8)(x - 6) = 0$
$x = -8, 6$
Can't have negative length so $x = 6$cm

3 $\frac{4}{x} + \frac{x+1}{3} = \frac{12}{3x} + \frac{x(x+1)}{3x} = \frac{12}{3x} + \frac{x^2 + x}{3x} = \frac{x^2 + x + 12}{3x}$

4 $PO = \frac{\sqrt{200}}{2}$ $RPT = 85.1°$ (1 d.p.)

5 $x = \pm\frac{5}{3}\sqrt{h}$ 6 $g = 197$

7 $\frac{y}{4} = \frac{2y + 3}{10}$ 8 $x < 3$
$y = 6$cm

9 $13m^2 - 26m + 26$ 10 58

11 a) 49.6° (1 d.p.)
b) 18.8cm (1 d.p.)