

# Fast Track 4

## Mathematics Workbook

W. Geldof

Written for Year 12 students studying NCEA Level 2 courses  
containing Algebra and Calculus

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2nd Edition



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## Statistics Database Available on Sigma's Website

In the Statistics chapter of this workbook you will be asked to investigate a database of information on New Zealand athletes. You can download the complete database from Sigma's website. You will require a computer with internet access and the microsoft spreadsheet program *Excel*.

Download instructions :

- 1] Go to Sigma's website at [www.sigmapublications.co.nz](http://www.sigmapublications.co.nz) and click on 'Secondary Maths Books' in the side panel menu.
- 2] Click on the cover picture of '*Fast Track 4*' at the bottom of this page.
- 3] This will take you to the '*Fast Track 4*' page. At the bottom of this page you will find the NZ Olympic Athletes spreadsheet listed.
- 4] To download the database, click on the spreadsheet.

## About this Book

### A Choice of Standards

This mathematics workbook '*Fast Track 4*' prepares students to pass NCEA Level 2 Achievement Standards with merit or excellence. The work is drawn from levels 6 and 7 of the New Zealand Mathematics and Statistics curriculum.

There are a total of 14 Achievement Standards available at Level 2 Mathematics. It is expected that a full-time Level 2 student will study between 5 and 7 of these standards (about 20-24 credits). The selected standards that students sit at this level must cover the three strands of the curriculum : Number and Algebra, Geometry and Statistics. Most schools will make this selection based on the ability and interests of their students.

Since no Level 2 student is expected to do all 14 standards, it would be a waste of money and paper to ask able maths students to purchase a workbook containing enough material to cover the full range of 14 standards. Therefore, after consultation with Heads of Maths departments, I have selected the nine most popular achievement standards being taught to the vast majority of *able* Level 2 maths students.

'*Fast Track 4*' provides revision work for the following Achievement Standards :

AS 2.1 - Coordinate Geometry .....	2 credits
AS 2.2 - Graphical Methods .....	4 credits
AS 2.3 - Sequences & Series .....	2 credits
AS 2.4 - Trigonometry .....	3 credits
AS 2.6 - Algebraic Methods .....	4 credits
AS 2.7 - Calculus Methods .....	5 credits
AS 2.9 - Statistical Inference .....	4 credits
AS 2.12 - Probability Methods .....	4 credits
AS 2.14 - Systems of Equations .....	2 credits

[black credits are internally assessed, orange credits indicate externally assessed standards.]

It is expected that students will use a range of technology throughout the year (graphing calculator, spreadsheet). This book includes some basic instructions for the *Casio fx9750 GII* graphic calculator and the *Microsoft Excel* spreadsheet. However, these explanations are brief and more detailed information can be found in the graphic calculator's instruction booklet and in *Excel's* help menu.

I wish you well with your studies this year.

*Wierze Geldof*