E Rolling Five Dice

When making a frequency table it is important that the result of each trial is recorded with **exactly one tally stroke**.
The result of any trial may not have two different outcomes.

1 Alan decides to complete an experiment where five ordinary dice are rolled together 60 times.
A trial will consist of rolling the five dice and recording how many of the dice show the same number.
For example, the set (2 2 2 4 5) gets a score of 3, because 3 of the dice show the same number.

a) What score would Alan give : i) (1 3 3 4 6) ..... ii) (1 5 5 5 5) .....

b) Alan needs to make decisions on how to record the results of some trials. His decisions may differ from other students.
For instance, Alan decides to give (1 2 4 5 6) a score of 1. His reason is : *'each number is different, it occurs only once.'*

i) Alan must make a decision about the outcome (4 4 4 6 6). Should it score a 3 or score a 2, or perhaps both?

Alan decides to give it a score of 3. What could be his reason? .....

.....

ii) How do you think he would record (1 1 2 2 6)? Say why. .....

.....

c) Alan was told that he should have defined **a trial** as *'rolling the five dice and recording the maximum number of these dice showing the same number'*. Explain why this definition would be better.

.....

.....

2 You are now going to complete this experiment using the definition in c) above.

a) List the set of possible outcomes for a trial. .....

b) What outcome do you think is the most likely? .....

c) What do you think is more likely; 3 of a kind or all numbers different? .....

3 Let the trials begin! You can collect your own data, or, if you are unable to get five dice, use Alan's results of 60 trials.

(1 2 2 4 5)	(1 1 2 6 6)	(2 3 3 3 5)	(2 2 5 5 6)	(1 3 4 4 5)	(1 1 1 4 6)	(2 3 3 6 6)	(1 3 4 5 6)	(2 3 5 6 6)
(1 3 3 4 6)	(1 2 3 4 6)	(1 3 4 5 5)	(2 2 3 4 6)	(2 3 3 3 5)	(1 2 5 5 6)	(1 1 4 4 6)	(1 2 3 5 6)	(2 3 4 4 6)
(1 1 3 5 6)	(2 2 3 3 5)	(3 3 5 6 6)	(1 1 3 4 6)	(2 2 3 5 6)	(1 1 2 5 5)	(2 2 3 3 5)	(1 1 1 6 6)	(2 2 3 6 6)
(1 1 2 4 6)	(1 2 3 5 6)	(1 2 3 3 5)	(1 3 4 5 6)	(2 3 3 4 5)	(1 3 3 3 5)	(2 4 5 5 6)	(1 3 4 5 6)	(1 4 5 6 6)
(2 2 3 6 6)	(1 2 2 4 6)	(4 4 4 4 5)	(1 2 3 5 5)	(1 3 4 4 5)	(1 2 2 3 5)	(1 3 3 4 6)	(2 3 3 4 5)	(1 2 2 3 3)
(2 2 3 4 4)	(2 3 3 3 6)	(1 5 5 6 6)	(1 2 3 4 6)	(1 2 3 3 6)	(2 3 4 5 5)	(3 3 5 5 6)	(4 5 5 5 6)	(1 1 4 4 5)
(3 3 4 5 6)	(2 4 4 5 6)	(3 3 3 4 6)	(1 1 4 4 6)	(1 2 3 4 4)	(1 4 6 6 6)			

a) Find the experimental probability distribution for the experiment.

b) Do your predictions of question 2 match the table?

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c) If the table shows that $P(5) = 0$, does that mean that it is impossible to roll 5 of the same? Explain your answer.

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.....

score	tally	frequency	experimental probability
	Total		

15 Theoretical Probability 2

A Tables for Combined Events

When an experiment involves two stages, for instance when two coins are tossed, two dice are rolled or two cards are drawn, then **tables** can be used to list the equally likely outcomes.

1 Two dice are rolled together.

a) Complete the table with all 36 equally likely outcomes (for example '4, 3' represents the outcome of a 4 on the first die and a 3 on the second die).

2nd / 1st	1	2	3	4	5	6
1						
2						
3						
4			*4, 3*			
5						
6						

outcomes on second die

outcomes on first die

b) The two numbers on the dice are added to get a total. Calculate the following probabilities.

i) P(total is 7) = ...

ii) P(total is 11) = ...

iii) P(total is greater than 7) = ...

2 A coin is tossed and a die is rolled; the combined outcomes are recorded.

a) Complete the table with the 12 equally likely outcomes.

coin / die	1	2	3	4	5	6
H	*H 1*	*H 2*				
T						

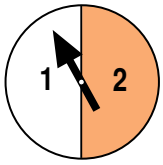
b) Calculate the following probabilities.

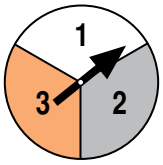
i) P(a head and a 6) = ...

ii) P(a head) = ...

iii) P(a head and a number less than 3) = ...

3





These two spinners are spun once each. The two numbers that come up are added to get a total.

a) Complete the table with all six equally likely totals.

2nd / 1st	1	2	3
1	*2*	*3*	
2			

b) Calculate the following probabilities :

i) P(total is 2) = ...

ii) P(total is 4) = ...

iii) P(total is more than 3) = ...

4 One card from hand A and one card from hand B is chosen at random.

hand A

hand B

a) Fill in the table with equally likely outcomes.

A / B	3	4
2	2 3	
3		

Calculate the probability of getting . . .

b) the same number twice. .....

c) the same suit twice. .....

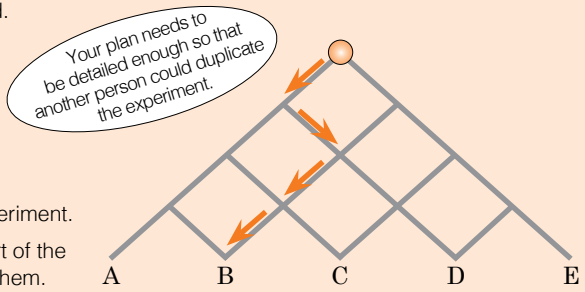
d) a total of at least 6. .....

e) a heart twice. .....

f) a red suit twice. .....

A Trials, Outcomes, Recording and Sources of Variation

Example : Write a plan for the experiment with the marble going down the grid.
Investigative question : What are the chances that the marble finishes at each of the points A, B, C, D, E?



For your plan you need to :

a) Discuss and define both **a trial** and the set of **possible outcomes** for a trial.

b) Decide on the **number of trials**.

c) **List the steps**, including your **recording system**, needed to perform the experiment.

d) You should consider the possible **sources of variation** that may exist as part of the collection of the data and possible actions that you could take to manage them. **Examples of possible sources of variation could be** : if the dice are not rolled fairly or if the spinner is weighted, the way a ball is thrown, the equipment that is used and the ability of the person taking the measurements.

Sources of Variation Categories

Sources of variation can be considered in five different categories :

1 **Natural variation** : this is the differences that occur due to nature, such as the variation in weights or head circumference between different people.

2 **Occasion to occasion variation** : this type of variation occurs when repeated measurements are taken over time. For example, a person's blood sugar levels will not be the same at different times of day.

3 **Measurement variation** : when taking a measurement, variation can occur in the method. For example, when measuring a height, it would be important to make sure the person being measured removes their shoes as this could add a different amount to each height due to the different sized soles and heels on shoes worn by different people.

4 **Induced variation** : different factors may affect the same measurement. For example, the price of fuel in two different petrol stations in different areas of the country may be affected by their location relative to the nearest port. They could also be part of different companies.

5 **Sampling variation** : this always occurs when a sample is taken. Unless the entire population of interest is measured, sampling variation will be present. This means that each time a different sample is taken, variation will occur in the individuals present in the sample and therefore the sample statistics will vary between samples.

Possible Plan :

a) A trial would consist of releasing the marble at the top of the grid and noting which letter it ends up at.
The set of possible outcomes would be : {A, B, C, D, E}.

b) There are five possible results and I need to have enough trials so there is a good chance that there is a reasonable number of marbles ending up at each letter. The more trials I have, the better my estimates will be but I also have to complete the experiment in a limited time frame. I think 60 trials will meet all these needs.

c) I will use the same marble each time and make sure it is released exactly above the starting point. When the marble reaches the bottom of the grid I will note the letter it ends up at. I will use a tally chart to record the number of times the marble ends up at each of the different letters. The tally chart will be used to make a frequency table that shows the experimental probability distribution. I will use the frequency table to graph the results.

d) Sources of variation. I will use the same marble each time, as a different marble could roll differently. I will release the marble from the same point each time which needs to exactly above C, otherwise it could be unfairly weighted towards one side.

As part of your plan, you could also include a prediction of what you expect to find.

29 Display the Experimental Data 2

A In the Long Run

Graphing the experimental probabilities at regular stages during your experiment helps when it comes to discussing patterns in your data (see pages 11-13).

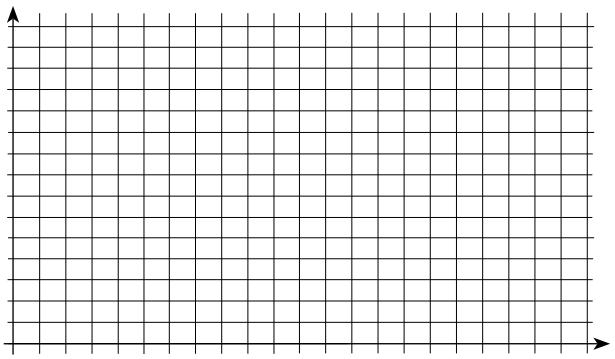
1 The following data is the number of levels built before the card house collapsed for 40 trials.

1	0	1	2	3
3	0	2	4	4
4	4	3	2	0
4	4	4	5	3
2	0	5	6	4
3	5	0	4	3
0	4	3	1	3
0	0	4	3	2

a) Complete the table showing the experimental probabilities for completing four levels before the card house collapses.

Nº of trials	Nº of 4s	P(4)
5	0	0
10	2	0.2
15	4	
20		
25		
30		
35		
40		

b) Graph the experimental probabilities after each group of five trials.



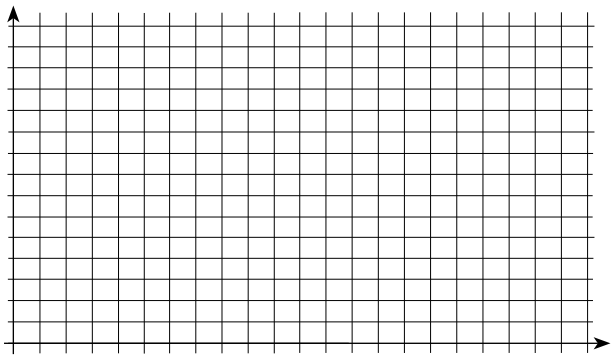
2a) The results of rolling two dice 60 times are shown on page 27, exercise A2. Record the differences in this table.

3	4	1	1	2	1

b) Complete the table showing the experimental probabilities of getting a difference greater than two.

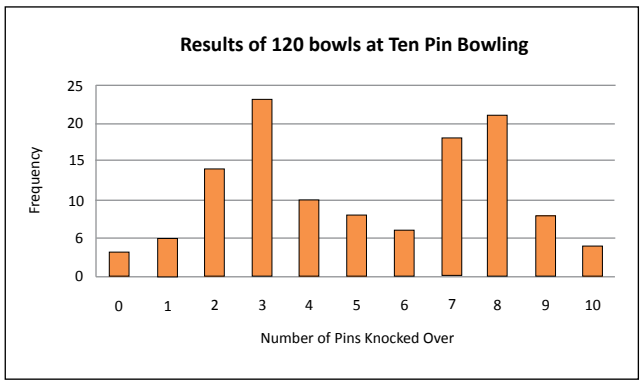
Nº of trials	Nº of rolls with a difference >2	P(>2)
6	2	0.33
12	2	0.17
18	4	
24		
30		
36		
42		
48		
54		
60		

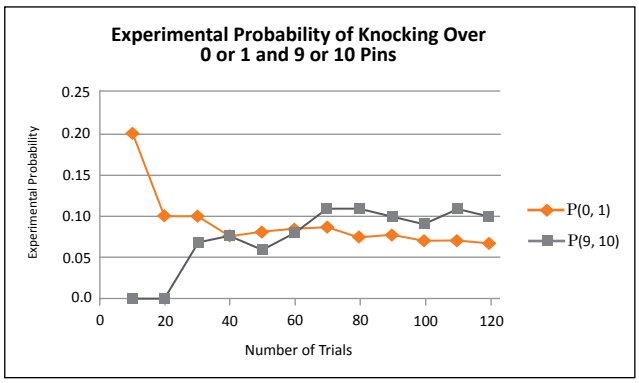
c) Graph the experimental probabilities after each group of six trials.



A Patterns - continued

6 Investigative Question : In ten pin bowling how does my chance of knocking over 9 or 10 pins compare with my chance of knocking only one pin over or missing all pins with one roll of a ball?





Patterns : .....

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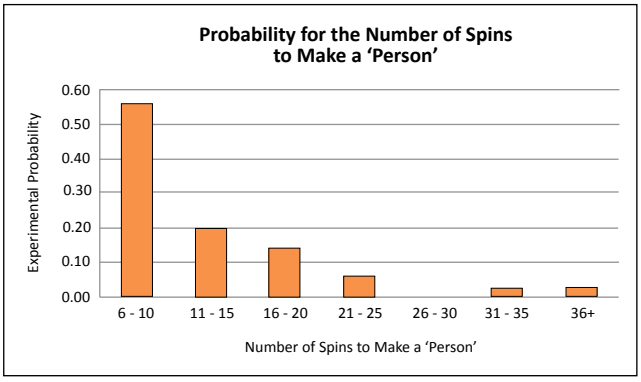
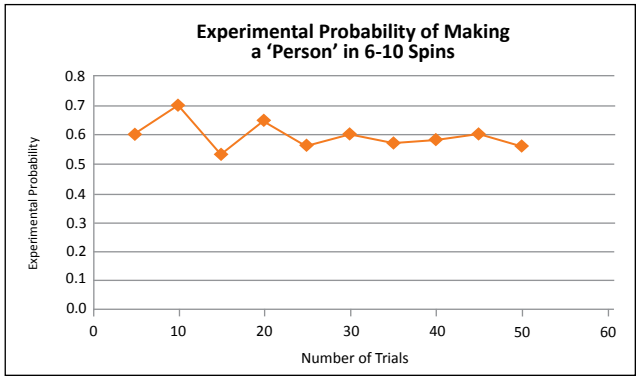
7 Investigative Question : In the *Make a Person* game what are the chances that the number of spins needed to a make a *person* is from 6 to 10, 11 to 15, 16 to 20, . . .?





Patterns : .....

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45 Practice Investigation 1

A Game for Matariki Festival

Nikau is in charge of making a game to raise money at his school's Matariki festival. His game involves dealing out ten cards and giving out prizes for certain numbers of "Hearts" among the ten cards. Customers must pay to play. Investigate an experimental probability distribution relating to the possible number of "Hearts" among the ten cards and suggest how Nikau could plan to give our prizes.

1 Pose your question :

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2 Plan your investigation :

a) Discuss and define both a trial and the set of possible outcomes for a trial :

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.....

b) Decide on the number of trials :

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.....

.....

c) List the steps, including your recording system, needed to perform your experiment :

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d) Possible sources of variation :

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.....

3 Gather your data by performing the experiment :

Pages 22-25 - Plan the Experiment - continued

Examples of possible plans.

A4 a) A trial consists of the two players playing snap, going through a deck of cards once and recording the number of snaps (pairs).
There are 26 pairs in a pack of cards so the set of possible outcomes for the number of snaps is any number from zero to 26.
b) Responses about the number of trials will vary.
c) The deck of cards needs to be well shuffled between each trial. For each trial the deck will be split exactly in half and then the two players will take one half each and play snap by turning their cards up on a pile in the middle of the table and recording how often, while going through the deck once, there is a snap (a pair). The tally chart needs to record all results up to 26, but it is not likely all will be used.
d) Possible sources of variation include the way the cards are shuffled SNAP.

A5 a) A trial consists of the person bowling a ball at the ten pins and recording the number of pins they knock over. On any bowl the player could knock over any number between 0 and 10 pins so the set of possible outcomes is {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
b) Responses about the number of trials will vary.
c) The player needs to try to hit as many pins as possible each time they bowl the ball. The number of pins that are knocked over will be recorded and transferred to a tally chart that records values from 0 to 10.
d) Possible sources of variation include the way the ball is bowled. Trying to make sure that the ball is bowled the same way each time. Using the same ball each time as heavier balls may be more likely to knock over more pins.

A6 a) A trial consists of counting the number of spins it takes to get a 'person'. The minimum number of spins that would be needed to make a 'person' would be six but there is no upper limit.
The set of possible outcomes is {6, 7, 8, . . .}.
b) Responses about the number of trials will vary.
c) Care needs to be taken to ensure that the spinner provides a random result each time. The sequence of body parts that result from spins will be recorded until there is one head, one body and four limbs.
The number of spins it takes to complete a 'person' will be collated in groups (6-10, 11-15, . . .) on a tally chart.
d) Possible sources of variation include the way the spinner is spun, making sure it is given a good spin and is able to spin freely each time. Possibly starting from a different position each time to ensure it doesn't end up at the same place every time.

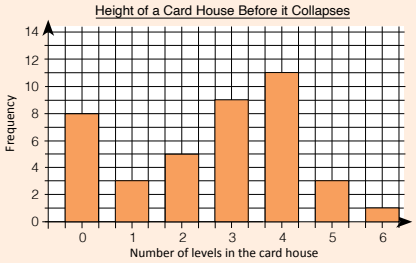
Pages 26-28 - Display the Experimental Data 1

Examples of Possible Displays :

A1 Display One

Nº levels	frequency	experimental probability
0	8	0.200
1	3	0.075
2	5	0.125
3	9	0.225
4	11	0.275
5	3	0.075
6	1	0.025

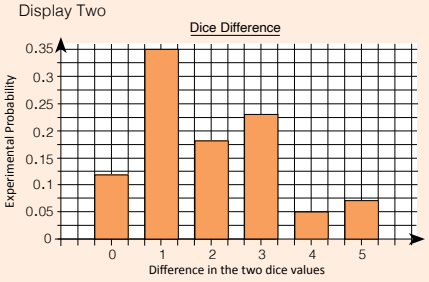
Display Two



A2 Display One

Difference	frequency	experimental probability
0	7	0.12
1	21	0.35
2	11	0.18
3	14	0.23
4	3	0.05
5	4	0.07

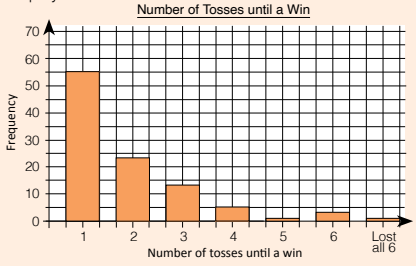
Display Two



A3 Display One

tosses	frequency	experimental probability
1	55	0.55
2	23	0.23
3	12	0.12
4	5	0.05
5	1	0.01
6	3	0.03
Lost all six	1	0.01

Display Two

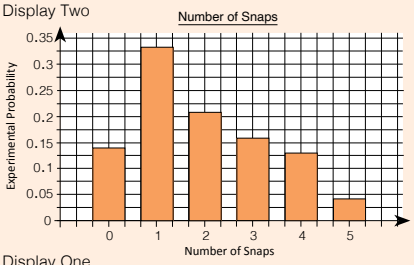


A4 Display One

Nº Snaps	frequency	experimental probability
0	11	0.14
1	26	0.33
2	17	0.21
3	13	0.16
4	10	0.13
5	3	0.04

Pages 26-28 - Display the Experimental Data 1 - continued

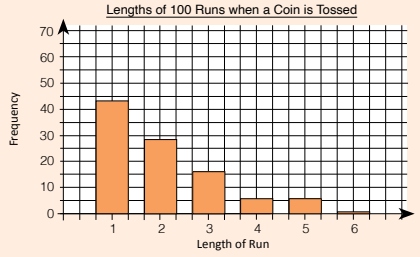
A4 Display Two



A5 Display One

Run length	frequency	experimental probability
1	43	0.43
2	28	0.28
3	16	0.16
4	6	0.06
5	6	0.06
6	1	0.01

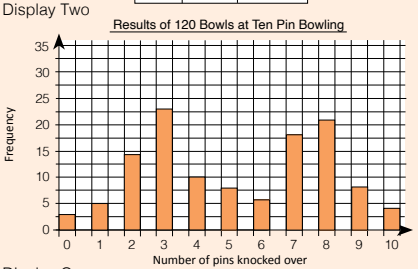
Display Two



A6 Display One

tosses	frequency	experimental probability
0	3	0.03
1	5	0.04
2	14	0.12
3	23	0.19
4	10	0.08
5	8	0.07
6	6	0.05
7	18	0.15
8	21	0.18
9	8	0.07
10	4	0.03

Display Two



A7 Display One

Nº Spins	frequency	experimental probability
6 - 10	28	0.56
11 - 15	10	0.20
16 - 20	7	0.14
21 - 25	3	0.06
26 - 30	0	0.00
31 - 35	1	0.02
36 +	1	0.02

Display Two