Integers 1

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Chapter 1 Number Knowledge

A Negative Numbers

Integers are whole numbers on either side of zero, including zero itself. -2 means '2 below zero'. – negative numbers — positive numbers -4 -3 -2 -1 0 1 2 3 4 5 6 On the numberline a number is larger than any number on its left and smaller than any number on its right. Example : Order these from smallest to largest : -8, 2, 6, -4. Answer: -8, -4, 2, 6. 1 В (Write down the integer. at A b) at B c) at C a) Circle the larger of the two numbers. 2 b) 0 -2 c) 2 -5 a) -3 4 3 Order these from smallest to largest. -5, 6, -8 a) 0. -500 -100, 200, b) 4 The temperature in Wanaka on Tuesday at 6 pm was 3°C. By midnight the temperature had dropped 4 degrees. What was the temperature at midnight? a) b) After midnight the temperature dropped another 2 degrees before it went up by 10 degrees, reaching the highest temperature on Wednesday at 2 pm. What was the highest temperature on Wednesday?

B Adding and Subtracting

Adding a positive number makes the original number larger. Adding a negative number makes the original number smaller.

	Examp	le :	Work these	e out b	y movi	ng o	n the n	umbe	erline.		
	++-		a) 2 + 5	(0	3+4		;) 3 + +	2	a) 4	· +	3 -+
	-7 -6	-5	-4 -3 -2	2 -1	0 1	2	3 4	5	6 7	8	9
	Working	g:a b c d) 2 + 5) ⁻ 3 + 4) 3 + ⁻ 2) ⁻ 4 + ⁻ 3	mean: mean: mean: mean:	s start a s start a s start a s start a	at 2 at -3 at 3 at 4,	, go u , go u , go d go d	p 5, p 4, own 2 own 3	ansv ansv , ansv , ansv	ver ver ver ver	7 1 1 7
1	Work	out.									
a)	3 +	6				b)	-4 +	5		••••	
c)	-6+	1				d)	2 +	4		••••	
e)	5 +	-2				f)	-7 +	-1		••••	
g)	-2+	8				h)	-3+	-2			

Subtracting a positive number makes the original number smaller. Subtracting a negative number makes the original number larger.

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Example :	Work these	out by m	noving or	n the numb	erline.
	a) 6 - 8	b) -2 -	-1 c)	32	d) -25
+ + + -7 -6 -5	-4 -3 -2	-1 0	1 2	3 4 5	6 7 8 9
Working :	a) 6 – 8 r b) [–] 2 – 1 r c) 3 – [–] 2 r d) [–] 2 – [–] 5 r	neans sta neans sta neans sta neans sta	art at 6, art at -2, art at 3, art at -2,	go down go down go up 2, go up 5,	3, answer ⁻ 2 1, answer ⁻ 3 answer 5 answer 3

2 Work out.

i)

Q

-5

a)	9-5	 b)	2 - 8	
c)	-3 - 5	 d)	-1-3	
e)	22	 f)	56	
g)	-13	 h)	-44	
i)	-2 - 8	 j)	6 - 7	

3 Altogether now. Work these out.

a)	8 + -4	 b)	-5 - 4	
c)	-9+6	 d)	10 - 8	
e)	37	 f)	-2 + 10	
g)	-82	 h)	0 + 6	
i)	4 + -7	 j)	2-9	

Conversions

A	Fraction - Decimal	B	Use the Calculator
	Example : Write $\frac{3}{4}$ as a decimal number. Working : $\frac{3}{4}$ can mean three quarters, but also, three divided by four. Use a calculator $3 \div 4 = 0.75$ Answer : $\frac{3}{4} = 0.75$		We can order fractions by writing them all in decimal form. Example : Order from smallest to largest $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{4}{7}$. Working : Decimal form: 0.666, 0.6, 0.625, 0.571. Order : $\frac{4}{7}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{2}{3}$.
1	Write these fractions as decimal numbers.	1	Circle the largest of these pairs.
a)	$\frac{1}{5}$ b) $\frac{7}{10}$	a)	$\begin{array}{ccc} \frac{3}{8} & \frac{7}{18} \\ 17 & 0 \end{array}$
c)	$\frac{3}{8}$ d) $\frac{35}{100}$	b)	$\frac{17}{23} \frac{2}{3} \qquad \dots \qquad $
e)	$\frac{9}{25}$ f) $\frac{13}{20}$	C)	<u>5</u> <u>9</u> 11 <u>20</u>
	When converting a fraction to a decimal we can end up with a screen full of digits. This pattern of digits will repeat indefinitely, we call it a recurring decimal . This is how we write recurring decimals : Examples : $\frac{1}{3} = 1 \div 3 = 0.3333 = 0.\overline{3}$ $\frac{1}{27} = 1 \div 27 = 0.037037 = 0.037$ $\frac{7}{29} = 7 \div 22 = 0.31818 = 0.318$	2 a)	Order these from smallest to largest. $\frac{3}{8}$, $\frac{1}{3}$, $\frac{2}{7}$, $\frac{3}{10}$ $\frac{7}{8}$, $\frac{8}{11}$, $\frac{11}{9}$
2	Write each fraction as a recurring decimal	0)	8' 9' 13' 11
2	$\frac{2}{2}$ b) $\frac{7}{2}$		3 18 4 34
a)	3 ······ b) 9 ·····	c)	$2\frac{1}{4}, \frac{10}{7}, 2\frac{1}{5}, \frac{10}{15}$
C)	27 d) <u>11</u>		
e)	$\frac{5}{24}$ f) $\frac{5}{44}$	0	
	To convert a decimal to a fraction we must remember how to read decimals. For instance, 0.45 can be read as forty-five hundredths. $0.45 = \frac{45}{400} = \frac{9}{90}$	3	Grandma has bought a large number of identical balls of wool. She finds that 5 balls of wool are needed to make 4 socks and that 9 balls of wool are needed to make 7 hats.
	[0.45 can also be read as 4 tenths and 5 hundredths. $0.45 = \frac{4}{10} + \frac{5}{100} = \frac{40}{100} + \frac{5}{100} = \frac{45}{100} = \frac{9}{20}$]	a)	How much wool - as a fraction of a complete ball - does grandma need to make
3	Write these decimals as fractions in simplest form.		i) 1 sock?
a)	0.8		ii) 1 hat?
b)	0.05	b)	Which garment needs more wool, a sock or a hat? First use a mental strategy to work his out, then check with a calculator.
c)	0.16		
d)	0.125		
e)	0.068		

Fractions and Decimals (23)

Simplifying Multiplications

Chapter 3

Expressions

A The Long and the Short of It

Guidelines for Simplifying Multiplications :

multiply numbers.

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- place number in front of letters.
- place letters in alphabetical order.
- leave out the times sign (x).

It is often a good idea to write a multiplication in long form first, then combine the terms to write the shortest possible expression

Examples : Simplify

Working :

a)	4y x 3		b) 5 b	x 2	2a				C)		3 p :	r×	20	1	
Wor	king :	a) b) c)	4y x 3 5b x 2a 3pr x 2q	= = =	4 5 3	X X X	y b p	X X X	3 2 r	x x	a 2	х	q	= = =	12y 10ab 6pqr

1	Write these	e in long form first, then sir	mplify.
a)	6p x 5		
b)	8 x 2w		
c)	2a x 6		
d)	4 x 2y		
e)	0 × 3z		
2a)	$3z \times 5w$		
b)	3a × 6b		
c)	$4y \times 4z$		
d)	ab x 4c		
e)	ac × 2bd		
	Remember	how to multiply integers : pos x pos = pos neg x neg = pos	neg x pos = neg pos x neg = neg
	Examples :	Simplify. a) $-4a \times 5b$	b) -2q x -3pr

a) $-4a \times 5b = -4 \times a \times 5 \times b = -20ab$

Note: a means 1a

b) $-2q \times -3pr = -2 \times q \times -3 \times p \times r = 6pqr$

B Lots of Letters

	Examples : S a)	Simplify 2a x 5a	b) 2a ²	× ⁻4a	c) a	b ² x 3a
	Working : a) b) c)	$2a \times 5a =$ $2a^2 \times -4a =$ $ab^2 \times 3a =$	2 × a 2 × a a × b	x 5 x a x a x ⁻ 4 x b x 3 >	= × a = < a =	10a2-8a33a2b2
1	Simplify.					
a)	4p x 5p					
b)	$\mathbf{q}^3 imes \mathbf{q}^4$					
c)	$y^2 \times 3y$					
d)	$2yz \times 3z$					
e)	$2p^3 \times p$					
f)	$2a^2 \times 3a^3$					
g)	$2ab^2 \times 2a^2b$					
h)	$3q^2r \times 7r^3$					
2	Simplify.					
a)	-2a x -3a					
b)	4p ² x -2					
c)	-у×Зу					
d)	$b^2c \times -c$					
e)	$-2q^3 \times 5q$					
f)	-6ab × -3a					

3	Write these in long form first, then simplify.	3	Simplify the	se without writing	the	long form first	
a)	-4b x 2a	a)	$a^3 \times a^2$		b)	$\mathbf{p}^5 \times \mathbf{p}$	
b)	6p x ⁻ 3	c)	2y x -3		d)	ab × a	
c)	⁻ 2w × ⁻ 5y	e)	-2b × -3b		f)	$c^2 \times 2ac$	
d)	⁻ a x 3b	g)	⁻a×⁻a		h)	3pq × 4p	
e)	$2z \times -3w \times 4y$	i)	$\mathbf{y}^5 \times \mathbf{y}^5$		j)	$3a^3 \times 4a^4$	
		k)	$ab^2 \times bc^2$		I)	-2p ² x -p	

Straight Lines

A Points on a Grid

A cartesian plane is a grid with two axes.

The horizontal axis is called \mathbf{x} -axis,

the vertical axis is called **y-axis**, the intersection is called the **origin**. Each point in this plane is defined by a pair of numbers known as its **coordinates**.

	у			D	
	-3-		\mapsto	¢r	
R *	-2-				
	-1-				
)			
-2 -1	-()	1 2	2 3	→ 3 X
-2 -1	1÷	s		Q	→ 3 x
-2 -1	1÷	s	1 2	ې و	> x

 \mathbf{S}

Т

Example :

Point P has x coordinate 2 and y coordinate 3. In short P is at (2, 3). Q is at (2, 2), R is at (2 . 2), and S at (0, 1). The origin is at (0, 0).

1

				у						
	X									\mathbf{s}
			I				С			
		,	<u> </u>	0	,					
	1 4 1 \	-	 2 		R	:	 2 	2	1	x
	E									
			T	2-	,	F				M

Write the coordinates under each letter.

X C O M E

(**...,3**, **..3**.) (....,)(....,)(....,)(....,)

F I R S

(....,)(....,)(....,)(....,)

2

				У	Ì					
				2						
	1 4 1	-	1 2 1		ľ	:	2	4	4 1	x
 	4	17	2			:	2		4	x
 	4	-:	2	2-		:			4	x

- a) Plot A at (4, 2), B at (1, -1), C at (0, 3). Draw triangle ABC.
- b) Plot P at (-5, 1), Q at (-5, -2), R at (-1, -2).
 Plot another point, S, such that PQRS is a rectangle.

The coordinates of S are (.....)

B Hidden Pictures

1



We will draw a bat by connecting points.

Start at (0, 3) and connect this with (-2, 4), then to (-1, 3), (-2, 2), (-3, 3), (-2, 1), (0, 1), (0, 3), (-1, 6), (5, 5), $(2, 4), (3, 3), (2, 3), (2, 2), (\frac{1}{2}, 2), (1, 1), (1, 0),$ (0, -1), (-2, 0), (-2, -1), (-3, -1), (-3, -3), (-4, -1), (-5, -4),(-5, 2), (-2, 1).Draw a face for the bat.

2 Draw your own picture. Write instructions so somebody else can copy it.

Start at (,), connect with	(,), ()
----------------------------	----	-------

(.....), (.....), (.....), (.....), (.....),

Coordinates



A Conversion Diagram

Diagram for converting units of volume.							
	x 1000	×	To convert from L to mL multiply by 1000.				
L K	÷ 1000	mL /	To convert from mL to L divide by 1000.				
Examples :	3.5 L 870 mL	= 3500 ml	L (3.5 x 1000 = 3500) (870 ÷ 1000 = 0.87)				

1 Convert.

a)	6 L	= mL	b) 4.8 L	= mL
c)	10.5 L	= mL	d) 0.32 L	= mL
e)	8000 mL	= L	f) 2600 mL	= L
g)	1950 mL	= L	h) 740 mL	= L



2 Convert to the unit shown.

a)	83 t	= kg	b)	2.4 g = mg
c)	0.75 kg	= g	d)	9000 kg = t
e)	500 g	= kg	f)	680 mg = g



3 Convert to the unit shown.

a)	96 cm	= mm	b)	4.8 m = cm
c)	8.5 km	= m	d)	0.04 m = mm
e)	607 cm	= m	f)	1800 m = km
g)	450 mm	= m	h)	75 mm = cm

B Solving Problems

In measurement problems there are situations where units have to be changed.

Example : A box contains 65 full cotton reels. Each reel weighs 28 g, the empty box weighs 150 g. Work out the total weight of the box with reels, giving your answer in kg.
Working : 65 x 28 g = 1820 g; 1820 g + 150 g = 1970 g

Answer ·	1 97 kr	1
	1.01 1.0	4

1 Wayne's drinking bottle can hold 600 mL of water. Wayne drinks 2 of these bottles every day of the working week and 3 each day of the weekend. How many litres of water does Wayne drink per year? (a year has 52 weeks).

A cardboard box is filled with tins of peaches as shown.
 Each tin of peaches weighs 260 g (this includes the tin).
 The box weighs 80 g.



- a) Calculate the weight of the cardboard box filled with tins, give your answer in kg.
- A shipping container takes a load of 1500 boxes of peaches.
 Calculate the weight of this load in tonnes.
- 3 Work out. a) 3.5 km + 500 m = km b) 53.8 cm + 25 mm = cm c) 1 L - 220 mL = mL
- 4 Amy's Christmas presents are stacked from longest to shortest. For each present select its length.



Transformations

Properties of Transformations



A	Thinking About Trans	formations							
1	Dylan says, 'In all four transformations objects and image are congruent.' Do you agree? Give a reason.								
0									
2	Esther says, "Performing a in its new position."	a translation is easy, I sele	ect just one point of the o	object, P , and find its image	e P'. Then I copy the shape				
	Can you think of a fast wa	y of performing an enlar	gement?						
3a)	An object is reflected in mi	rrorline \mathbf{m} . Point \mathbf{B} and its	reflection \mathbf{B}^{I} are exactly	v the same point. What does	s this tell you about point B ?				
b)	A triangle is rotated about	t centre (° One vertex st	ave in exactly the same	a place. What does this tal					
0)									
4	Triangle ABC and its image would be possible.	ge, triangle $A^{l}B^{l}C^{l}$, are at	exactly the same posit	tion. Give details of 3 trans	formations for which this				
	i)								
	ii)								
	iii)								
B	One Object - Four Ima	ages							
1									
	object	image 1	image 2	 image 3	image 4				
	Four different transformati transformation and give the	ions have been performe ne details.	ed on the purple triang	e. For each image write do	own the type of				
a)	image 1 is the result of								
b)	image 2 is the result of								
c)	image 3 is the result of								
d)	image 4 is the result of								

Compass, Protractor and Ruler

Chapter 7 Construction

A Construct and Measure

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1 Construct exact diagrams in the space below, then take measurements as required.



A D

Ρ

Angle Rules





A The Hypotenuse In a right angled triangle, the side facing the right angle is called the hypotenuse. A Example : In Δ ABC, which side is the hypotenuse? Answer : Side AB. In each of these triangles colour the hypotenuse red. 1 В С A is the right angle of a right angled triangle. We will check Pythagoras' rule for different lengths of sides. Example : With pencil, draw a line from B to C, measure side \overline{BC} . In $\triangle ABC$, side $\overline{BC} = 5.4$ cm, $\overline{AB} = 2$ cm, $\overline{AC} = 5$ cm. Checking : \overline{BC} is the hypotenuse. 2 Lucy says, "The hypotenuse is the longest side of a right Is 5.4^2 equal to $2^2 + 5^2$? angled triangle." Check Lucy's statement by measuring the $5.4^2 = 29.16$ or 29 to the nearest whole. right angled triangles above. $2^{2} + 5^{2} = 4 + 25 = 29$. Pythagoras' rule is correct. Do you agree with Lucy? Choose B and C in other positions on the lines. 1 Pythagoras was a mathematician who lived 2000 years ago in Measure \overline{AB} , \overline{AC} and \overline{BC} , then check Pythagoras' rule. Greece. He discovered that in every right angled triangle the square of the hypotenuse equals a) First choice : the sum of the squares of the $\overline{\mathrm{BC}}$ = cm, $\overline{\mathrm{AB}}$ = cm, $\overline{\mathrm{AC}}$ = cm other two sides, or $c^2 = a^2 + b^2$. h $\overline{\mathrm{BC}}^2$ = Example : Write the rule of Pythagoras for this right angled triangle. $\overline{AB}^2 + \overline{AC}^2 = \dots$ Working: q is the hypotenuse so $q^2 = p^2 + r^2$. Comment : b) Second choice : 3a) Which side in this triangle is \overline{BC} = cm, \overline{AB} = cm, \overline{AC} = cm the hypotenuse? r $\overline{\mathrm{BC}}^2$ = Write down Pythagoras' rule for this triangle. b) $\overline{AB}^2 + \overline{AC}^2 = \dots$ Comment : Write down Pythagoras' rule for each of these triangles. 4 c) Third choice : a) b) $\overline{\mathrm{BC}}$ = cm, $\overline{\mathrm{AB}}$ = cm, $\overline{\mathrm{AC}}$ = cm z $\overline{\mathrm{BC}}^2$ = $\overline{AB}^2 + \overline{AC}^2 = \dots$ v Comment :

B Your Choice

Organising Data



A Graphing Paired Data

In some investigations the data comes in pairs. Paired data is graphed on a grid with two axes and each data point is plotted with a cross.

Paired data can be found in two types of investigations :

- Investigating a relationship between two variables.
 Example: We investigate the relationship between height and weight of girls at our school.
 Each girl in our sample of 20 comes with a pair of measurements : (height in cm, weight in kg), so the graph consists of 20 plotted points.
 In the graph showing a relationship we do not connect the crosses.
 The graph is called a scatter plot or a scatter graph.
- 2] Investigating changes in a variable over time.
 Example: We investigate changes in the New Zealand population from 1950 to 2010.
 If the variable 'population size of NZ' is recorded at 10 year intervals starting at 1950, then we get a list of 7 pairs (year, population), so the graph consists of 7 plotted points.
 In the graph showing changes over time we do connect the crosses with straight lines.
 The graph is called a line graph or a time series graph.
- 1 Mr Wright reads the electricity meter on the last day of each month. The list shows the units he used last year.

Jan a) Plot a graph for the data. Feb Mar b) What is the name of the graph? Apr May Or Jun c) i) What reading seems to be out of line? July Aug ii) What could be the reason for this? Sep Oct Nov Dec



Weight vs Height (girls)



2 Brooke investigates the relationship between length of femur (thigh bone) and overall height of girls in Year 10. She took measurements from a sample of 12 girls from her Year 10 class.

Name	Kate	Lucy	Chloe	Amy	Amber	Molly	Olive	Tessa	Isla	Emily	Leah	Sophie
length of femur (cm)	45	49	48	46	50	51	48	44	49	47	47	52
height (cm)	162	169	166	164	168	171	164	161	167	162	166	171

month

units

180

250

50

360

480

500

610

750

590

480

310

220

- a) Plot the points on this grid.
- b) What is the name of the graph?
- c) Brooke forgot to include her own measurements. She is 167 cm tall and her femur measures 49 cm. There is a small problem with plotting Brooke's measurements. What is the problem and how would you solve it?



Experimental Probability 2

Chapter 11 Experimental Probability

A My Experiment

 There are six steps in conducting an experiment:
 1 - Define the problem
 2 - Set up the experiment
 3 - Do the trials

 4 - Draw conclusions
 5 - Think about improvements
 6 - Test the conclusion with more trials

A coin when tossed has two possible outcomes, a head or a tail. If the coin is fair it should land on heads about half the time. We don't need an experiment to find this out.

Find an item which has two possible outcomes when dropped (for example a drawing pin, bottle top, buttered slice of toast). You are going to do an experiment to calculate the probability of landing on one of the two positions.

Step 1 - Define the problem (Complete these sentences.)	
In this experiment I will toss a	I expect that there are two possible outcomes,
either	or
I want to find the probability that it lands on	
Step 2 - Setting up the experiment(Describe how you will do endHow many trials will there be	ach trial. Will you roll, flick or drop the thing? ce? Draw up a tally table.)
For each trial I will	
I will do each trial times.	······
I will keep a tally of the outcomes in this tally table.	
Step 3 - Do the trials (Make sure you count the trials.)	
Step 4 - Draw conclusions (Describe what happened.)	
In this experiment I did trials and found that	
The proportion of times my object landed on	is
If I do this experiment another times I expect to g	jet
Step 5 - Think about improvements (Did anything go wrong during	g the trials? Should you have done things differently?)
Step 6 - Test the conclusion with more trials (You could do your o	wn experiment again or swap with another student.)
The experiment was done again and it was found that this time	the proportion was
That is different / about the same. (cross one out.)	
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