Points on a Line

A No Graphs Needed

Many question about points and lines can be answered without drawing the graph.

Examples :

a) A = (4, 3), B = ($^-$4, $^-$2), C = ($^-$2, 0)
Which of these points lie on line $y = \frac{1}{2}x + 1$?

b) Point Q is the point where the line y = 4 crosses the line y = 2x - 3. What are the coordinates of Q?

Working :

a) Substitute x and y values into the equation $y = \frac{1}{2}x + 1$
Check whether the equation is true.

Point A : x = 4, y = 3	$3 = \frac{1}{2} \times 4 + 1$	True
Point B : x = $^-$4, y = $^-$2	$^-2 \neq \frac{1}{2} \times {}^-4 + 1$	False
Point C : x = $^-$2, y = 0	$0 = \frac{1}{2} \times {}^-2 + 1$	True

So points A and C are on the line.

b) Since Q is on the line y = 4, its y-coordinate is 4.
Since Q is also on the line y = 2x - 3 then,
$4 = 2x - 3 \Leftrightarrow 7 = 2x \Leftrightarrow x = 3\frac{1}{2}$.
Therefore Q = ($3\frac{1}{2}$, 4)

1 A = (0, $^-$2), B = (8, 0), C = (8, $^-$2)
Which of these points are on the line $y = \frac{1}{4}x - 2$?

2 Point S is the intersection of lines x = 4 and y = x + 6.
What are the coordinates of S?

3 T is the point where the line y = $^-$2 crosses the line y = 3x – 1. What are the coordinates of T?

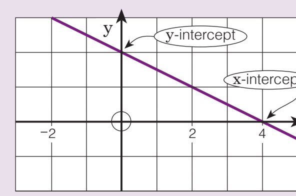
4 A = (2, 1), B = (6, 7).
Which of these points is on line $y = \frac{3}{2}x - 2$, as well as line y = 2x – 5?

B Crossing the Axes

The point where a line crosses the x-axis is called the x-intercept.
The coordinates of an x-intercept are always of the form (a, 0).

The point where a line crosses the y-axis is called the y-intercept.
The coordinates of a y-intercept are always of the form (0, b).

Example :
For the line $y = {}^-\frac{1}{2}x + 2$
the x-intercept is (4, 0) and the y-intercept is (0, 2).



The intercepts of a line can be found without drawing a graph.

Example : Find the intercepts of y = 3x - 1

Working :

x-intercept (a, 0)	y-intercept (0, b)
y = 0 ; y = 3x - 1	x = 0 ; y = 3x - 1
3x - 1 = 0	y = 3 x 0 - 1
3x = 1	y = $^-$1
$x = \frac{1}{3}$	
x-intercept = ($\frac{1}{3}$, 0)	y-intercept = (0, $^-$1)

1 Find the intercepts of these graphs.

a) y = 4x + 6 ; intercepts (…….., ……..), (…….., ……..)

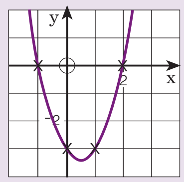
b) $y = \frac{2}{3}x + 4$; intercepts (…….., ……..), (…….., ……..)

c) $y = \frac{3x - 6}{2}$; intercepts (…….., ……..), (…….., ……..)

C Writing an Equation Using the X-Intercepts

If the x-intercepts ($x = p$, $x = q$) are known, an equation of the form $y = a(x - p)(x - q)$ can be formed.

Example : Write an equation for the parabola shown in this graph.



Working : We can easily read off the position of the x-intercepts, they are at $x = {}^-1$ and $x = 2$. So the equation has the form $y = a(x + 1)(x - 2)$.

To work out the value of a, we need to read off the coordinates of one other point on the parabola. This point does not have to be the vertex, any point will do. Since (0, ⁻3) is on the graph, we replace x with 0 and y with ⁻3 which enables us to calculate a :

$^-3 = a(0 + 1)(0 - 2) \iff {}^-3 = a \times {}^-2 \iff a = \frac{^-3}{^-2} = 1.5$

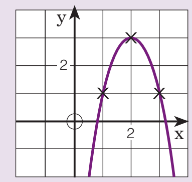
Hence the equation is $y = 1.5(x + 1)(x - 2)$.

Check by graphing this function on your GC.

D Writing an Equation Using the Vertex

If the coordinates (h, k) of the vertex are known, an equation of the form $y = a(x - h)^2 + k$ can be formed.

Example : Write an equation for the parabola shown in this graph.



Working : The position of the x-intercepts is not clear, but we can read off the coordinates of the vertex, it is at (2, 3). So the equation has the form $y = a(x - 2)^2 + 3$.

With just one other point on the graph the value of a can be found.

Since (3, 1) is on the graph, we replace x with 3 and y with 1 :
$1 = a(3 - 2)^2 + 3 \iff 1 = 1a + 3 \iff a = {}^-2$.

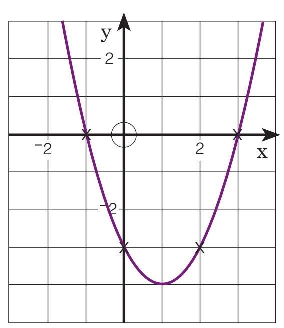
Hence the equation is $y = {}^-2(x - 2)^2 + 3$.

Check by graphing this function on your GC.

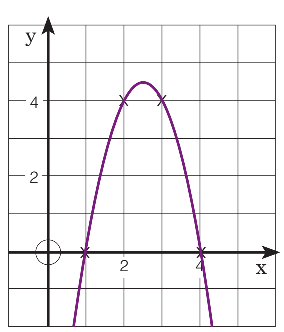
C

1 Write an equation for each graph.

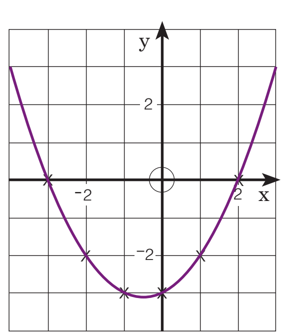
a) ....



b) ....



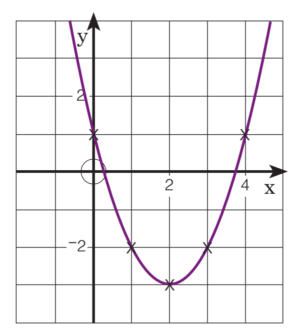
c) ....



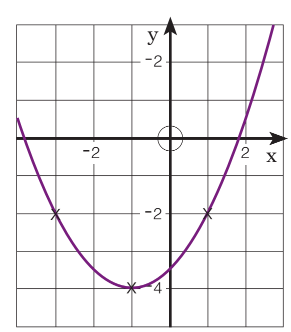
D

1 Write an equation for each graph.

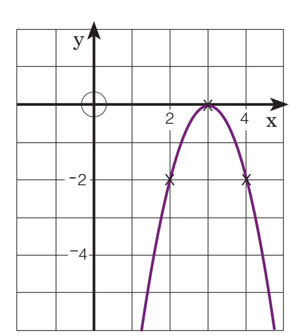
a) ....



b) ....



c) ....



A Solve Using a Spreadsheet

The following problem will be solved with the help of technology, firstly using a Spreadsheet then using a Graphing Calculator (see B).

Problem : A long rubber hose, 99 metres long, must be cut into pieces in such a way that each piece is 40 cm longer than the previous piece. The first piece is 30 cm. How many pieces will be cut? How long will the longest piece be?

n	term	sum
1	30	30
2	70	100
3	110	210
4	150	360

Analysing the Problem :

The sequence of hose lengths (in cm) is 30, 70, 110, 150, . . .
This is an AP with $t_1 = 30$ and $d = 40$. We know that the sum of the terms, $S_n = 9900$.
We are asked to work out n and t_n.

Working : We will generate the 3 columns on a spreadsheet, extend them until we find the number 9900 in the sum column. Two possible methods are shown.

Method A : Set up a spreadsheet using the following facts.

a) The first N equals 1, each following N value is 1 more than the previous.

b) The first TERM value is 30, each following TERM value is 40 more than the previous.

c) The first SUM value equals 30, each time the SUM value is increased by the just calculated TERM value.

◇	A	B	C
1	N	TERM	SUM
2	1	30	30
3	=A2+1	=B2+40	=C2+B3
4	=A3+1	=B3+40	=C3+B4

Fill Down

Method B : Set up a spreadsheet using the formulas for t_n and S_n.

a) The first N equals 1, each following N value is 1 more than the previous.

b) The all terms are found by $t_n = t_1 + (n - 1)d$, in this case term = 30 + (n − 1) x 40.

c) The sums are found by $S_n = \frac{n}{2}(t_1 + t_n)$, in this case sum = $\frac{n}{2}$(30 + term).

◇	A	B	C
1	N	TERM	SUM
2	1	=30+(A2-1)*40	=A2/2*(30+B2)
3	=A2+1	=30+(A3-1)*40	=A3/2*(30+B3)
4	=A3+1	=30+(A4-1)*40	=A4/2*(30+B4)

Fill Down

Solving the Problem :

Inspect the columns : Down at ROW 23, you will find SUM = 9900, which goes with N = 22 and TERM = 870.

Solution : There will be 22 pieces, the largest will be 870 cm long.

B Solve Using a Graphing Calculator

We know $S_n = 9900$. Since we don't know t_n, we will use the sum formula $S_n = \frac{n}{2}(2t_1 + (n - 1)d)$.

Substituting all known values gives : $9900 = \frac{n}{2}(2 \times 30 + (n - 1)40)$.

This last equation has only one variable: n. Use the equation solver on the GC; choose EQUA from the Main Menu, Select F3 : Solver.

Type in the equation using X for the letter n. 9900 = X ÷ 2 x (60 + (X − 1) x 40) press EXE.
Choose any initial guess* for X, e.g. X = 6 and press F6 : SOLV.
Result : for X = 22 the left hand side and right hand side of the equation are both 9900.

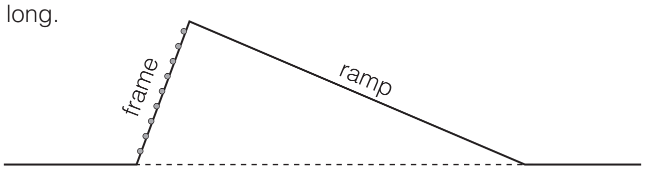
Back to the problem :
For $n = 22$, the sum is 9900 as required, so there are 22 pieces and the last piece of rubber hose has length $t_{22} = 30 + 21 \times 40 = 870$ cm.

* Note : The Equation Solver on your GC uses a very clever 'guess, check and improve' method to solve equations. You always have to provide a first guess. If an equation has two possible solutions the calculator could come up with 'the other' solution. For instance, if your initial guess for X were negative, then the GC would have given the answer X = ⁻22.5, which is an impossible solution for n. Always check whether the solution given by the calculator makes sense. If not, try another initial value.

A The Great Outdoors

In these questions diagrams are drawn without measurements. Read the question and write the given details on the diagram.

1 In an adventure playground there is a triangular structure with a sloping ramp and a climbing frame on the opposite side. The climbing frame is 3.5 m long and makes an angle of 80˚ with the horizontal ground, the ramp is 8.3 m long.



What angle does the ramp make with the ground?

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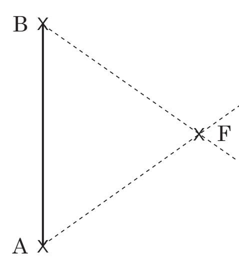
2 A and B represent two forest lookout towers, B is 4.5 km directly north of A. A fire is sighted on a bearing N32˚E of A and S55˚E of B.

How close is the fire to tower B?



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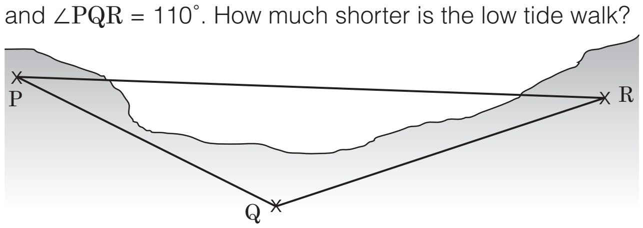
3 A tramper arrives at an estuary (at P). At low tide he can walk straight from P to R, but at high tide he has to walk on the beach via Q which is 3.1 km from P; ∠RPQ = 40˚ and ∠PQR = 110˚. How much shorter is the low tide walk?



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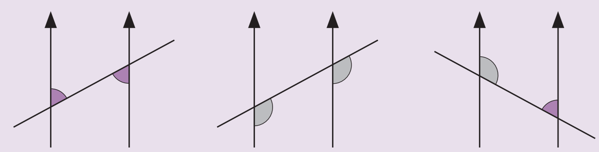
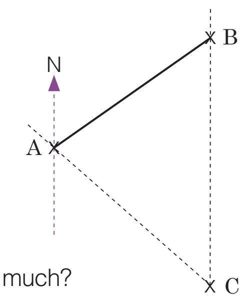
B Bearings

When solving a problem involving bearings, we start by drawing lines pointing north. North pointing lines are parallel.
Remember the angle rules when dealing with parallel lines :

1) Alternating angles on parallel lines are equal.
2) Corresponding angles on parallel lines are equal.
3) Co-interior angles on parallel lines add to 180˚.



1 From city A, the bearing to town B is N48˚E and to town C is S75˚E. Town B is 12 km north of town C.



a) Use angle rules to find all angle sizes in ΔABC. Write these in the diagram.

b) Which town is closer to city A? By how much?

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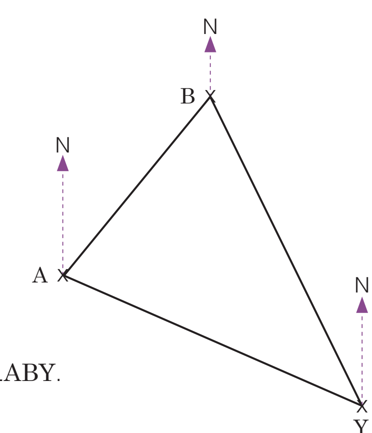
2 The distance between lighthouses A and B is 6.8 km. The bearing to B from A is N35˚E. The skipper on yacht Y measures the bearing to A as N72˚W and to B as N28˚W.



a) Calculate ∠AYB and ∠ABY.

....

....

b) Calculate the distance of the yacht to lighthouse A.

....

....

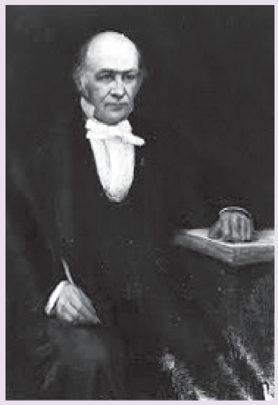
....

80 Visiting All Nodes Once

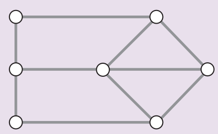
A Hamilton Paths and Circuits

We investigated traversability, i.e. passing over every edge of the network just once, the number of times each node is visited is not important.
Now we will look at visiting every node in the network just once without having to go over all the edges.

Irish mathematician William Rowan Hamilton working in the 19th century, investigated networks for which it is possible to visit every node just once. A possible path going through every node just once is called a Hamilton path. If the path also returns to its starting point it is called a Hamilton circuit. In that case the first node is visited twice.

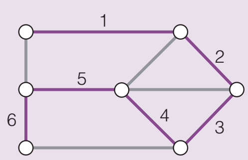


Example : Find a Hamilton path in this network. Colour the edges travelled. Is it a circuit?

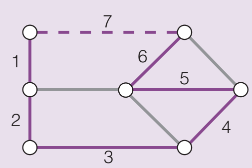


Working : Here are two possible answers (there are more).

i)



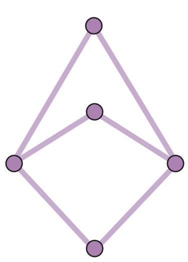
ii)



i) shows a Hamilton path, while ii) is a Hamilton circuit.

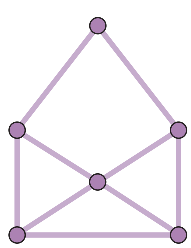
1 If possible find a Hamilton path or circuit in each of these networks. Colour and number the edges of your path. Check whether the path you found is a circuit.

a)



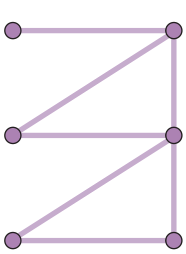
path / circuit / impossible

b)



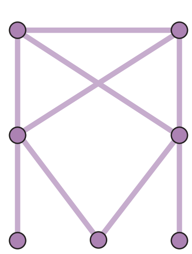
path / circuit / impossible

c)



path / circuit / impossible

d)



path / circuit / impossible

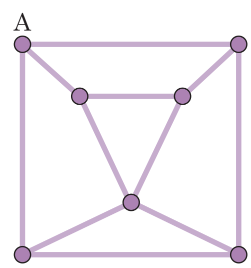
B Hunting for Hamilton Paths and Circuits

Unfortunately there is no rule telling us whether a Hamilton path or circuit exists in a network and where to find the starting and or finishing node. However, here are some properties to look for in the network when hunting for a Hamilton path or circuit.

- If the network has n nodes, then a Hamilton path traverses $(n - 1)$ edges, a circuit traverses n edges.
- For a *circuit* there must not be any nodes of degree 1. For a *path* nodes of degree 1 must be starting / finishing nodes, which restricts the options.
- If there are nodes of degree 2 then both edges incident to it must be in the path / circuit.
- No smaller circuit can be contained in any Hamilton circuit (because the start / end node of the smaller circuit would have to be visited twice).

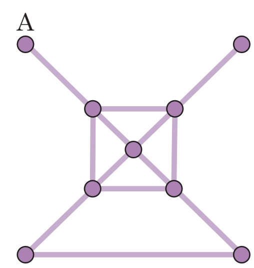
1 Study each network and hunt for a Hamilton *circuit* starting at A. If that is not possible, look for a Hamilton *path* starting at A. Colour and number the edges travelled.

a)



path / circuit / impossible

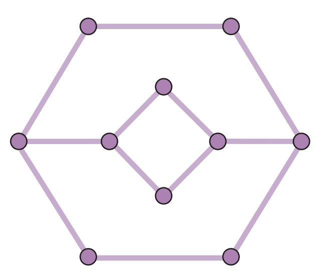
b)



path / circuit / impossible

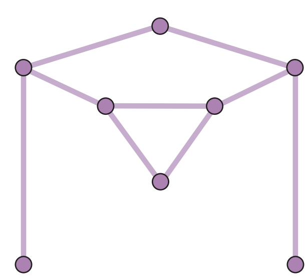
2 Hunt again for circuits or paths, starting at a node of your choice.

a)



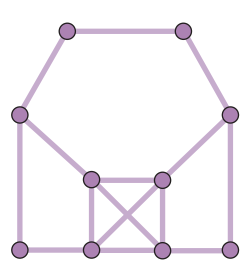
path / circuit / impossible

b)



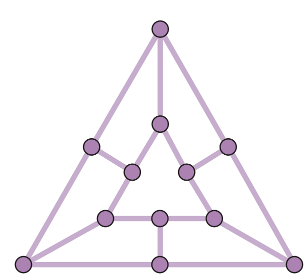
path / circuit / impossible

c)



path / circuit / impossible

d)



path / circuit / impossible

Sample Statistics 1

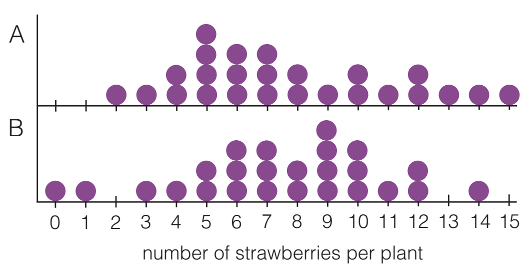
The variation of the scores within the sample is analysed by looking at the shape of the distribution and by calculating statistical measures. There are measures of central tendency like mean and median which give information about the scores at the centre. There are also measures of spread or dispersion like range, interquartile range and standard deviation which give information about how the data is spread out from the centre.

On the next three pages we will use formulas to work out statistical measures. The graphing calculator can do the work for us (see page 99).

A Calculating Median & Quartiles

The median is the middle value of the ordered set of numerical data.
The lower quartile (LQ) is the middle value of the lower half, while the upper quartile (UQ) is the middle value of the upper half of the data.
The range (R) = highest score minus lowest score.
The interquartile range (IQR) = UQ minus LQ.

1 A grower of strawberries is taking one systematic random sample of 25 plants of plot A and another of 25 plants from plot B. He counts the number of strawberries per plant.



Calculate the statistics of each group.	Plot A	Plot B
Median	....	....
Lower quartile	....	....
Upper quartile	....	....
Interquartile range	....	....
Range	....	....

2 The frequency table shows the ages of 25 gymnasts. Calculate median, quartiles, range and interquartile range.

age	f
16	5
17	6
18	5
19	3
20	3
21	3

Med .... LQ ....

UQ .... R ....

IQR ....

B Box and Whisker Plot

1 This stem and leaf plot shows weights of 35 punnets of strawberries.

23	5 7 8
24	2 4 4 6 7 8 8 9
25	0 2 2 4 5 5 6 6 7 8
26	0 1 2 4 5 6 7 7 9
27	1 2 3 3 4

key 23|5 = 235 grams

a) Find lowest and highest weight.

....

b) Find LQ, Med and UQ.

....

....

c) Draw a box and whisker plot for the weights.

d) Write a comment describing key features of the distribution of the punnet weights.

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AS 2.8 - Test : Part A

Suggestion : This part of the practice test could be done with small groups of students working together. When finished the various groups could compare their lists of objectives.

A Defining the Objectives

Scenario : Your PE teacher has asked you to design a questionnaire that would enable him or her to find out whether students at your school have sufficient fluid intake.

1a) What is the survey objective? ....

....

b) Who is the sponsor? ....

c) Describe the target population. ....

d) Decide how the survey will be conducted. ....

....

2 Research the topic of hydration (the 'Hydration Notes' on page 110 could help). Then formulate 3 or 4 research objectives.

i) ....

ii) ....

iii) ....

iv) ....

3 For each research question write down a set of detailed research questions.

a) Detailed objectives for i) ....

....

....

....

....

b) Detailed objectives for ii) ....

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....

c) Detailed objectives for iii) ....

....

....

....

....

d) Detailed objectives for iv) ....

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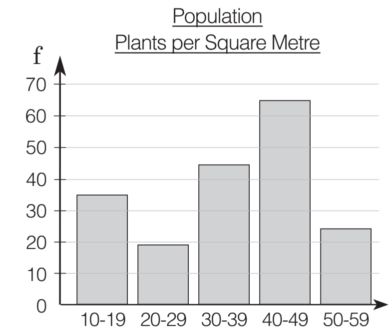
....

128 Variability Due to Sampling Method

A point estimate is a statistic, calculated from a random sample, that is used as an approximate value for a population parameter. For instance, the sample mean $\overline{x}$ is used as a point estimate for the population mean μ. The shape of the distribution of the sample is used to deduce the shape of the distribution of the population. This process of estimation is called statistical inference. By carefully selecting our sampling method and by taking steps to minimise non-sampling errors, we are unlikely to end up with a freak sample and more likely to get a *reliable* sample. A sample is said to be reliable if point estimates don't vary too much if another sample was taken.

A With or Without Strata

When there are two or more distinct subgroups in a population, a stratified sample or quota sample is expected to represent the population better. We will investigate.



Med = 39 LQ = 24 UQ = 45
μ = 35.3 σ = 13.0

38	47	45	30	43	38	42	49	24
50	33	53	39	51	45	32	44	34
45	56	36	51	43	37	34	49	42
32	43	14	22	12	16	24	15	13
51	57	12	23	15	12	18	10	11
46	35	40	39	40	47	38	41	28
37	53	42	50	32	43	45	35	41
20	15	10	16	12	39	56	46	37
21	18	24	19	20	46	33	52	44
25	48	43	48	42	54	40	45	30
41	53	32	37	41	44	37	39	54
52	36	44	41	38	46	31	42	44
46	49	17	23	16	14	23	14	10
38	55	13	24	15	17	22	17	16
51	42	39	45	41	33	40	49	32
40	37	56	47	20	18	50	35	41
31	44	33	47	18	12	43	48	41
39	50	52	40	20	23	38	52	39
43	34	35	40	17	19	46	44	27
48	44	42	38	15	12	55	44	54
45	25	33	43	12	11	31	36	42

1 A small plot of land, 9 metres wide and 21 metres long, has been sown in clover. The seeds have germinated and you can see little plants. Although the plot looks nice and green, there are patches where the soil is showing through. These patches are fenced off. The farmer wants to know the average number of plants per m^2. The population in this case is *'number of plants per square metre on 189 m^2 of land'*.

The diagram on the right shows the simulated situation, where the population is known. The purple coloured squares are the sparsely covered patches. There are 50 purple squares.

a) First take a random sample of at least 30 squares. This is not a stratified sample, so you must ignore the colour of the squares. How will you sample?

....

....

Place the results in the stem and leaf plot.

1 |
2 |
3 |
4 |
5 |

b) Calculate sample statistics.

Med ....

LQ ....

UQ ....

$\overline{x}$ ....

s ....

2a) Now we will take a stratified sample. How many squares should be selected from

from the sparsely covered patches? ....

b) Take two random samples, first of the bare patches, then of the lush area. Place the results together in one stem and leaf plot.

1 |
2 |
3 |
4 |
5 |

c) Calculate sample statistics.

Med ....

LQ ....

UQ ....

$\overline{x}$ ....

s ....

3 Compare the two samples with each other and with the population. Is stratified sampling an improvement? Comment.

....

....

Analyse, Conclude, Reflect 2

A The Effect of Tuition and Encouragement

Hypothesis : Small group tuition and encouragement has a positive effect on the development of bowling skills.

Two groups of students from an Intermediate School in Wellington were used in an experiment using a tenpin bowling alley. The two groups were set up properly using random allocation and efforts were made to equalise them. Each group consisted of 32 students.

Both groups started by watching a video where the basics of tenpin bowling were explained. The control group (group C) did not get any more help from supervisors. The treatment group (group T) had supervisors who gave them helpful hints and encouragement while they were playing.

The groups were not in the bowling alley together; the group with tuition went bowling first, while the others went to visit Te Papa Museum.

The game scores are shown in this stem and leaf plot.

Group C	Stem	Group T
9 9 7 6 6 5	2	7 8
9 7 7 6 5 5 4 0	3	2 4 4 5 7 9
9 8 8 4 4 3 2	4	1 1 3 4 4 6 6 8
8 8 7 6 5 5 2 0	5	3 3 5 6 6 7 8
8 5 3	6	0 2 5 5 5 6 7 9
	7	3

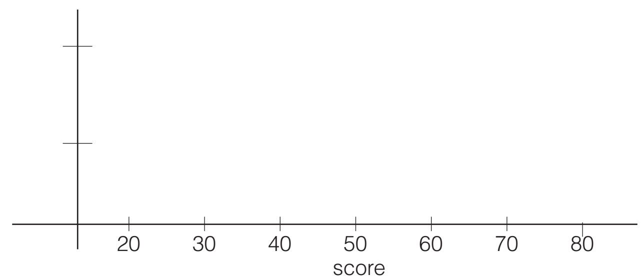
key : 2|7 = 27

B Analyse the Data

1 Calculate statistical measures.

	mean	st. dev	LQ	Med	UQ
Group C					
Group T					

2 Draw box and whisker plots.



3 What do you notice when comparing the groups?

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C Conclusion and Reflection

1a) Write a conclusion.

b) Reflect on the experiment. Think of external factors that could have influenced the results.

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C Evaluation of the Survey Method

1 List the variables (and measures) used in the survey.

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Remember : (see page 95)
Faulty or inconsistent measuring methods, environmental issues, or timing of the survey can cause *non-sampling errors*. Non-sampling errors have the potential to cause bias in surveys.
When collecting data, sources of non-sampling errors should be identified and neutralised if possible.

2 The data was collected by observation and stop watch. What sources of non-sampling errors were anticipated and dealt with in the survey?

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D Evaluation of the Analysis

1 Study Table 1a) and 1b) in the report.

a) How many females washed their hands (with or without soap?) ....

b) Of the females who washed their hands, what percentage washed with soap? ....

c) Of all females in the sample, what percentage washed their hands with soap and dried their hands.

....

d) What percentage of all males and females in the sample, washed their hands with soap and dried them?

....

2 The proportions calculated in 1c) and 1d) are not reported. Would these results be helpful? Give a reason for your answer.

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3 Comment on how effective the presentation of the results is.

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Introduction to Tree Diagrams

A Bushes

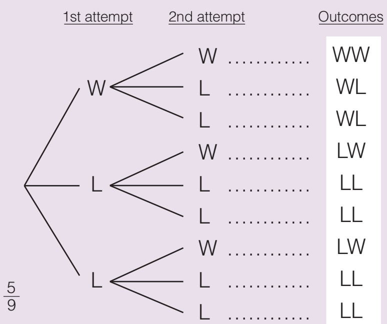
A tree diagram is a tool for solving problems involving probability.

Example : Sophie is playing a game in which the chance of winning is 1 in 3. Sophie plays the game twice.

a) Show the equally likely outcomes in a tree diagram.
b) Calculate the probability that Sophie wins just once.
c) Calculate the probability she wins at least once.

Answer :

a) In this diagram W stands for *win*, L stands for *loss*.
There are 9 equally likely outcomes



b) $P(\text{WL or LW}) = \frac{4}{9}$

c) $P(\text{WW or WL or LW}) = \frac{5}{9}$

1 Study the tree diagram in the example and calculate these.

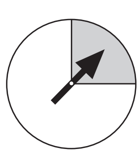
a) The probability that Sophie wins first time.

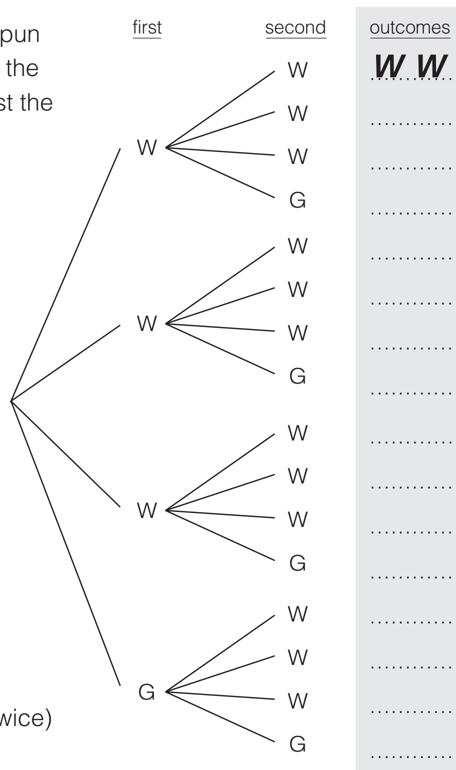
P(win first time) = …………………

b) The probability that Sophie wins both times.

P(win both times) = …………………

2a) This spinner is spun twice. Complete the tree diagram. List the 16 equally likely outcomes.





Calculate

b) P(same shade twice)

…………………

c) P(at least once white) …………………

B Trimmed Branches

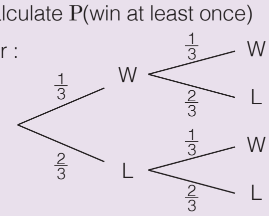
Tree diagrams could get cumbersome if there are too many branches. It is possible to *simplify* the tree diagram by placing probabilities on the branches.

Example : We take the same example as in A to show how to simplify the tree diagram. The chance of winning a game is 1 in 3. Sophie plays the game twice.

a) Draw a tree diagram, show the outcomes, each with their probability of happening.
b) Calculate P(win once)
c) Calculate P(win at least once)

Answer :

a)



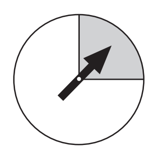
outcome	probability
W W	$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
W L	$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
L W	$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$
L L	$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

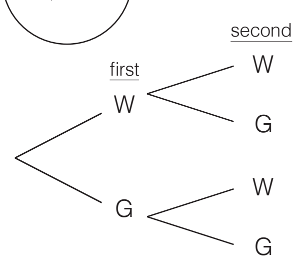
b) $P(\text{WL or LW}) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

c) $P(\text{WW or WL or LW}) = \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{5}{9}$

1 This spinner is spun twice (see exercise A 2).

a) Draw a simplified tree diagram, fill in the table.

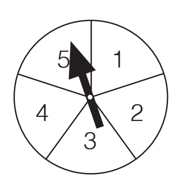


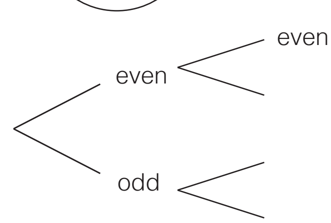


outcome	probability
……….	……………………….
……….	……………………….
……….	……………………….
……….	……………………….

b) Calculate P(different shades) ………………………………….

2a) This spinner is spun twice.
Finish the tree diagram and the table.





outcome	probability
……….	……………………….
……….	……………………….
……….	……………………….
……….	……………………….

Calculate

b) P(both times odd) ……………………………………...

c) P(once odd and once even) ……………………………….

Simulation Experiment 1

A Free Shots

1 In the game of basketball, a player can be granted up to 3 free shots, taken in succession, to make up for a foul against them. Joel has practised taking shots from behind the free throw line. About 1 in 4 of his attempts get in, this means for each shot the probability of scoring is $\frac{1}{4}$.

Your task is to simulate the situation of Joel taking 3 free shots at the basket. How many shots get in?

a) We can simulate the probability of scoring at each attempt by generating random numbers. Explain.

b) Explain what needs to be done for each complete trial in the simulation experiment.

c) How many trials will you do?

B Gathering Data

1 Carry out the experiment using random numbers. Record the outcome of each trial. Use this space to show your data.

SHOTS			score out of 3
1	2	3	

SHOTS			score out of 3
1	2	3	

C Summarise

1 On the right, draw a table that summarises the results.

2 What is the most likely outcome when Joel is awarded 3 free shots? ....

3 Calculate the experimental probability that Joel scores at least once when he is awarded 3 free shots.

A Using Substitution

The graphing method can only be used if both equations are written in the form $y = f(x)$. What if one equation is written in the form $y = f(x)$, but the other is not?
The method used to solve a mixed set is called the *substitution method* : one whole equation is being substituted into the other.

Example : Solve simultaneously. $y = 3x - 1$ (1)
$x + 2y = 12$ (2)

Working : Equation (1) tells us that y equals $3x - 1$, so we can replace the y in equation (2) by $3x - 1$.

Then equation (2) changes from $x + 2y = 12$
to $x + 2(3x - 1) = 12$.

Once an equation has just one variable, we can solve it algebraically. Start by expanding the brackets, then tidy up :

$$x + 2(3x - 1) = 12$$
$$x + 6x - 2 = 12$$
$$7x = 14$$
$$x = 2$$

Now we still need to calculate the value of y.
Work as follows : select one of the two original equations and replace x with 2.
Using (1) $y = 3x - 1$, we get $y = 3 \times 2 - 1 = 5$.

Solution : $x = 2$ and $y = 5$

1 Solve simultaneously.

a) $\left.\begin{array}{l} y = 2x + 3 \\ x + 4y = 30 \end{array}\right\}$

b) $\left.\begin{array}{l} y = 4 - 2x \\ 3x - 2y = 27 \end{array}\right\}$

c) $\left.\begin{array}{l} y = \frac{2}{3}x - 1 \\ x - y + 2 = 0 \end{array}\right\}$

B Word Problems

Many word problems can be solved with the substitution method.

Example :
A room is 4.2 metres longer than it is wide. The perimeter of the room is 29.6 metres. What are the dimensions of the room?

Working :
Let L be the length of the room and W the width (both in metres).
Given is $L = W + 4.2$ and also $2L + 2W = 29.6$.
We can substitute the first equation into the second and solve :

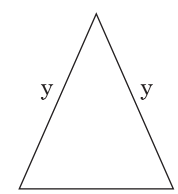
$2L + 2W = 29.6$ becomes

$$2(W + 4.2) + 2W = 29.6$$
$$2W + 8.4 + 2W = 29.6$$
$$4W = 21.2$$
$$W = 5.3$$

Since $L = W + 4.2$, then $L = 5.3 + 4.2$, $L = 9.5$

Solution : The length is 9.5 m, the width 5.3 m.

1 In an isosceles triangle the equal sides are 2.4 cm longer than the base. The perimeter is 9.6 cm. Calculate the length of the sides in this triangle.



2 Yvette runs 5 days and Zoe runs on 6 days every week. Yvette's daily run is just 3 km short of double Zoe's daily run. In one week Zoe and Yvette run a combined distance of 105 km. How far does each girl run per day? Complete the working :

Let Z be the distance (in km) of Zoe's daily run, and Y the distance of Yvette's daily run.

Then ***Y = 2Z –***

Also ***5Y +*** **=**