Integers, Powers and Roots

Chapter 1 Number Knowledge

A Order of Operations

4

Remember the order of operations when you do calculations. Work out brackets first, then powers, then x or \div , then + or -. Be aware that x and \div have equal priority, just like + and -.

Examples : Calculate.

a) 5 - 3 + 7 b) $3(8 - 10)^2$ c) $\frac{-32 + 8}{-3 \times 4}$ Working: a) 5 - 3 + 7 = 8 + 7 = 15b) $3(8 - 10)^2 = 3 \times (-2)^2 = 3 \times -2 \times -2 = 12$ c) $\frac{-32 + 8}{-3 \times 4} = \frac{-24}{-12} = 2$

1 Calculate these.

15 - 8 + -10 a) b) 32 ÷ ⁻4 x 2 -6x-8-2 C) d) -2 + 5 x -3 **-**3 + 5² e) f) $2^4 \div 4^2$ g) $(3 - 5)^3$ h) $3(2+6)^2$ $\frac{-3(4-8)}{-2+-1}$ i) $\frac{-12}{-6}$ + (2 - 5)³ j) $6 + 3(-8 - 4) \div 9$ k) 1) 4(-10 + 3) + 5(8 - -6)

B Roots

	The square root, written as $\sqrt{}$, is the reverse of squaring. The cube root, written as $\sqrt[3]{}$, is the reverse of cubing.
	Examples : Calculate. a) $\sqrt{81}$ b) $\sqrt[3]{125}$
	Working: a) $\sqrt{81} = 9$, because $9^2 = 81$
	b) $\sqrt[3]{125} = 5$, because $5^3 = 125$
1	Work out these roots without the use of a calculator.
a)	√64 b) √121
c)	$\sqrt[3]{64}$ d) $\sqrt[3]{-8}$
 2	Fill in the missing numbers.
a)	$\sqrt{1.1.1.1} = 15$ b) $\sqrt{1.1.1.1} = 24$
 c)	$\sqrt[3]{} = 6$ d) $\sqrt[3]{} = 10$
 3	Show how you would estimate $\sqrt[3]{200}$.
	3
 4	Explain why $\sqrt{-64}$ can't be found, but $\sqrt[3]{-64}$ can.

O A Pattern

1	Complete these tables and use them to understand the meaning of negative powers. b) power fraction decima								decimal				
a)	2 ⁵	2 ⁴	2 ³	2 ²	2 ¹	2 ⁰	2 ⁻¹	2 ⁻²			2 ⁻¹	<u>1</u> 2	0.5
	32	16									2 ⁻²		
											2 ⁻³		
	$\div 2 \div 2$												
2	2 Calculate a) 3^2 b) 3^1 c) 3^0 d) 3^{-1} e) 3^{-2}												
0													
3	3 Write as a fraction and as a decimal												
a)	a) 5 ⁻¹ b) 10 ⁻¹ c) 10 ⁻²												
	-			5 /// O O	D 1 <i>i</i> :				utalat ta an da a t	4 -1 11	due al de la composición de		

Fraction Arithmetic 1

Chapter 2 Fraction Arithmetic

A Adding and Subtracting

We can add or subtract fractions when they have the same denominator. If they are not the same we must rename them.

Examples :	Work out a) $\frac{3}{8} + \frac{7}{8}$ b) $4\frac{1}{5} - 1\frac{4}{5}$
	c) $\frac{3}{4} - \frac{1}{3}$ d) $1\frac{1}{2} + 3\frac{3}{5}$
Working :	a) $\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$
	b) $4\frac{1}{5} - 1\frac{4}{5} = (4 - 1) + (\frac{1}{5} - \frac{4}{5}) = 3 - \frac{3}{5} = 2\frac{2}{5}$
	c) $\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$
	d) $1\frac{1}{2} + 3\frac{3}{5} = 1\frac{5}{10} + 3\frac{6}{10} = 4\frac{11}{10} = 5\frac{1}{10}$

1 Calculate.

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a)	$\frac{5}{9} + \frac{2}{3}$	
b)	$\frac{4}{5} - \frac{3}{10}$	
c)	$4\frac{1}{3} - 2\frac{2}{3}$	
d)	$5\frac{3}{5} + 6\frac{1}{2}$	
e)	$\frac{5}{6} - \frac{3}{4}$	
f)	$\frac{4}{9} + \frac{1}{5}$	
g)	$1\frac{2}{3} + 2\frac{5}{12}$	
h)	$4\frac{4}{9} + 2\frac{5}{6}$	
i)	$9\frac{3}{4} - 5\frac{5}{7}$	
j)	$8\frac{2}{5} - 3\frac{5}{8}$	
2	Calculate.	
a)	$\frac{3}{4} + \frac{2}{3} - \frac{5}{6}$	
b)	$3\frac{5}{6} + 1\frac{2}{3}$ -	2 1/2
c)	$1\frac{1}{2} - 3\frac{5}{9} +$	2 <u>1</u> 3
d)	$5\frac{1}{5} - 2\frac{3}{4} -$	<u>2</u> 15

B Multiplying

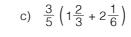
1	Study the diagram, then complete the	1 whole		
	sentence: Since $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$,	<u>1</u> 12		
	then $\frac{3}{4}$ of $\frac{1}{3}$ =			
	and $\frac{3}{4}$ of $\frac{2}{3}$ =	$\frac{1}{3}$		

Since the word of can be replaced by x we found
$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ which simplifies to $\frac{1}{2}$.
When multiplying fractions, we multiply numerators and
denominators. We can simplify the fractions as we go.
Examples : Multiply a) $\frac{3}{8} \times \frac{4}{5}$ b) $2\frac{2}{5} \times 1\frac{1}{3}$
Working: a) $\frac{3}{8} \times \frac{4}{5} = \frac{3 \times \frac{1}{2}}{2^{-8} \times 5} = \frac{3}{10}$
b) $2\frac{2}{5} \times 1\frac{1}{3} = \frac{12}{5} \times \frac{4}{3} = \frac{4}{5} \times \frac{4}{5} = \frac{16}{5} = 3\frac{1}{5}$

2 Multiply and simplify the answer.

a)	$\frac{4}{9} \times \frac{6}{7}$	
b)	$\frac{3}{8}$ x 10	
c)	$\frac{5}{8} \times \frac{4}{15}$	
d)	$\frac{9}{10} \times 1\frac{3}{7}$	
e)	$3\frac{1}{3} \times 2\frac{2}{5}$	

3 Calculate. a) $\frac{4}{5} \times \frac{3}{4} \times \frac{5}{6}$ b) $1\frac{1}{2} \times \frac{4}{7} \times 2\frac{1}{3}$



.....



Brackets

Factorising Expressions 1

Sometimes there is more than one way to factorise an expression.

or $2y \frac{y + 2}{2y^2 + 4y} = 2y(y + 2)$

The best answer is the one with the largest common factor.

 $2y^2 + 4y = 2(y^2 + 2y)$

 $2y^2 + 4y = y(2y + 4)$

B Highest Common Factor

Example : Factorise $2y^2 + 4y$ Working : $x y^2 2y$ Either 2 2y² 4y

or y

So $2y^2 + 4y = 2y(y + 2)$

 $\begin{array}{c|c} x & 2y & 4 \\ \hline y & 2y^2 & 4y \end{array}$

.

A The Opposite of Expanding

Factorise means write as a product, with brackets.

Example :	Factorise $4x + 20$
Working :	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Example :	Factorise $y^2 - 2y$
Working :	x y $^{-2}$ y $y^2 - 2y$ $y^2 - 2y = y(y - 2)$

1 Factorise.

a)	6a + 18 b) 4b + 20	1 Write down the largest common factor of these pairs.
,	х х	a) 42, 28 b) 108, 144
	6a 18 4b 20	c) 6a, $3a^2$ d) $15p$, $10p^2$
		e) $p^2 q$, $p q^2$ f) $6 a^2 b^3 \!\!, 9 a b^2$
c)	$5w - 35$ d) $y^2 + 4y$	
	x x	2 Factorise by taking out the largest common factor.
	$5w$ -35 y^2 4y	a) 2a ² + 4a
		b) ⁻ 3x ² - 12x
e)	$a^2 + a$ f) $w^2 - 3w$	c) 5b ² - 5b
		d) $4w^2 + 2w$
		e) 6y ² - 3y
		f) 4p ² - 8p
2	These have negative numbers in front of the brackets!	g) 10x ² + 15x
a)	⁻ 3a - 6 b) ⁻ 4x + 12	h) [−] 6b ² + 4b
		i) $y^4 + y^3$
		j) $x^2y + xy^2$
c)	⁻6w + 24 d) ⁻2y - 18	k) $8x^2 - 12x^3$
		3 The surface area of a cylinder is found with this formula : Surface area = $2\pi r^2 + 2\pi rh$ Eva factorised the area as $\pi(2r^2 + 2rh)$,
3	Do the working on your own paper when you factorise these.	
a)	$3x - 21$ b) $p^2 + 5p$	Who has the right answer? Explain.
c)	-4a + 32 d) -5y - 35	
e)	$x^2 - 6x$ f) $-2w + 24$	



A At a Constant Rate

1 After a birthday candle was lit, it got shorter over time as shown in the graph.



- a) How tall was the birthday candle when it was lit?
- b) Fill in the table showing the height (h) of the candle t minutes after it was lit.

t (min)	0	2	4	6	8
h (mm)					

c) i) How does **the table** show that the candle burned up at a constant rate?

	,	ow does the graph show the constant rate?	at the	candle burned up at
)		ct the correct formula giving lle after ${f t}$ minutes. (circle or		height (\mathbf{h}) of the
	А	h = 80 - 5t	В	h = 80 - 10t
	С	h = 80 + 5t	D	h = 80 + 10t

A straight line graph shows a situation with a constant rate of change.

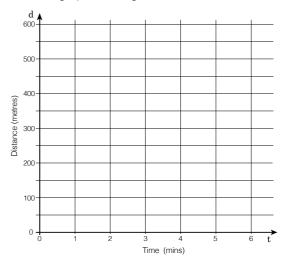
d

B At a Constant Speed

- 1 Amy and Ben went for a 600 metre run in the park.
- a) Amy ran the 600 m distance in 5 minutes and at a constant speed.
 - The graph showing the distance (d, in metres) ran by Amy t minutes after she started should be a straight line. How do we know?
 - ii) Explain why the points (0, 0) and (5, 600) must be on the graph.

.....

iii) Draw the graph on the grid.



- 2 The distance Ben covered on the track, t minutes after he started is given by the formula : d = 150t
- a) How far down the track was Ben after 2 minutes.
- b) Fill in the table and draw the graph of Ben's run in the park in the grid above.

t (min)	0	1	2	3	4
d (m)					

c) Who ran faster - Ben or Amy? How do the lines in the graph show this?

Measuring & Estimating

A More Problems

 A motor launch has a fuel tank with a capacity of 330 litres which allows it to travel a distance of 800 sea miles. It costs \$383 to fill the tank. At cruising speed the launch uses 2¹/₂ litres of fuel every hour.

Calculate . . .

- a) the cost of fuel per litreb) the amount of fuel used per sea mile.
- c) the cruising speed of the launch. [Hint : Use answer (b)]
-
- 2 Liquid Gold is a concentrated fertiliser and must be diluted to a spray at the rate of 250 mL per 10 L of water. How many litres of fertiliser spray will the standard 3.5 L container of Liquid Gold make?



3 A cylindrical tank containing 100 L of water has sprung a leak at the bottom of the tank and is losing water at a steady rate of 350 mL every minute. The height of the water level drops 8 cm every half hour.

How much water has the tank lost when the water level has dropped half a metre?

 c) A path cuts diagonally across the park. How long is the path
 in real life?

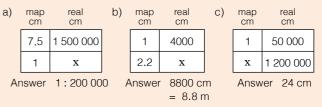
B Scale

Maps and plans are a representation on paper of a real situation. The **scale** is a ratio showing how much bigger the real situation is.

Examples :

- a) Work out the scale, if 7.5 cm on the map represents 15 km.
- b) Work out the real distance, if at a scale 1 : 4000 a distance is measured as 2.2 cm.
- c) The distance between two mountains is 12 km. How would this distance be represented on a map with scale 1 : 50 000?

Working : Use ratio tables.



1 Complete this table. (Draw ratio tables on scrap paper.)

	scale	distance on map	real distance
a)		3 cm	6 m
b)		5 cm	2 km
C)	1 : 300	18 mm	
d)	1 : 50 000	3.4 cm	
e)	1 : 200		4.6 m
f)	1 : 4 000 000		60 km

2 A rectangular park is 450 m long and 270 m wide.

a) Lee started a scale diagram of the park. She drew the length of the park 6 cm long. What scale did she use?

.....

b) Finish the scale diagram.

Using Rates 2

59)

Rotation

A Rotation

80

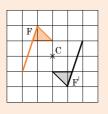
A **rotation** turns the object anti-clockwise through a certain angle about a fixed point. This point is called the **centre of rotation**.

Example :

Rotate flag F 180° about centre C.

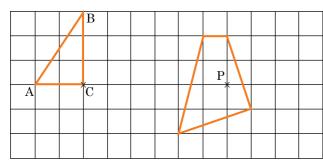
Working :

Copy the original flag F onto tracing paper, also copy C and draw a line going up from C like this : $\begin{array}{c} \downarrow \\ \times \\ C \end{array}$

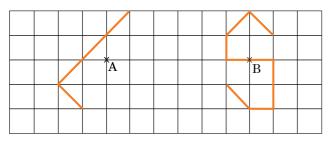


Place your pencil on top of the x on the tracing paper, turn the paper until the line points down $\stackrel{\text{X}}{}$ (that means you turned 180°). Copy the flag back onto your book. Label it \mathbf{F}^{I} .

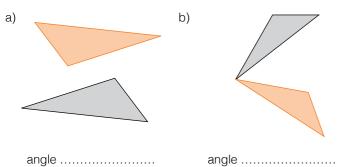
- 1a) Rotate the $\triangle ABC$ 180° about centre C.
- b) Rotate the quadrilateral 180° about centre P.



- 2a) Rotate the letter L 90° about centre A.
- b) Rotate the letter S 270° about centre B.



3 An orange triangle has been rotated to give a grey triangle. Work out the position of the centre C, mark it with a cross. Also work out the angle of rotation.



B Symmetry

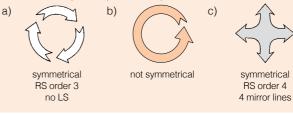
A shape has **line symmetry** (LS) if it has one or more axes of symmetry (mirror lines).

A shape has **rotational symmetry** (RS) if it fits on top of itself more than once during one full turn. The order of rotational symmetry = number of fits.

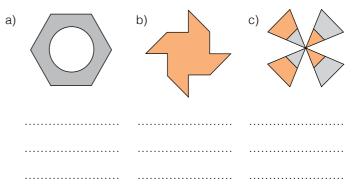
A shape is called **symmetrical** if it has line symmetry or rotational symmetry.

Example :

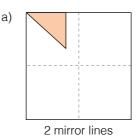
Describe the symmetry in these symbols.

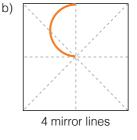


1 Describe the symmetry in these shapes.

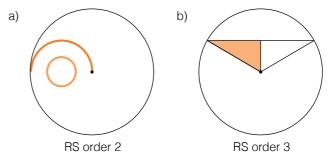


2 Finish these two patterns in such a way that the dotted lines are mirror lines.





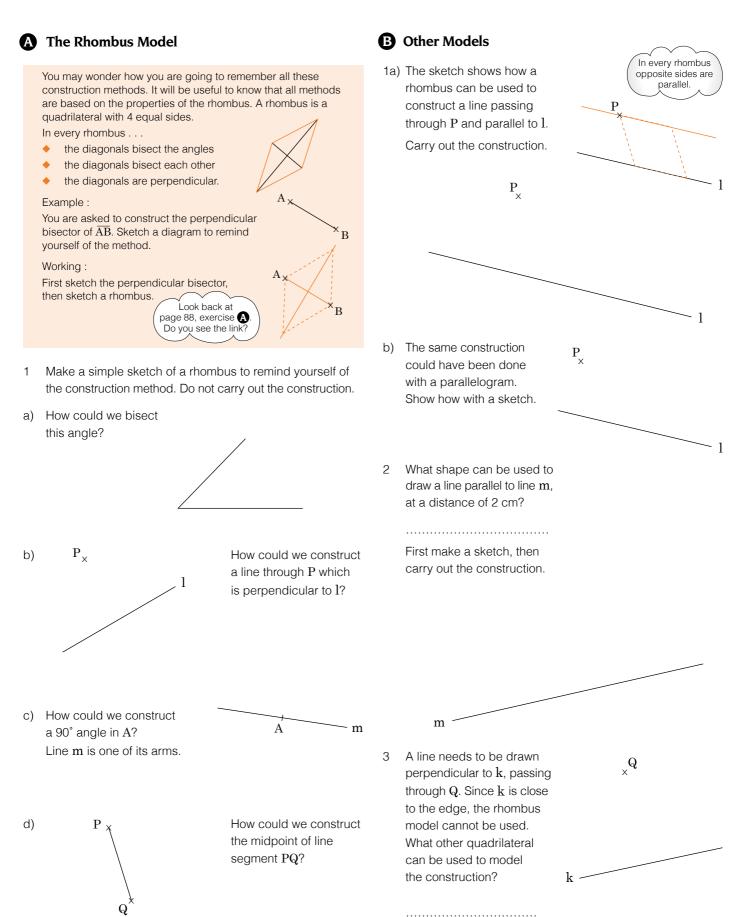
3 Finish these two patterns in such a way that they have rotational symmetry.



Constructions

Understanding Constructions

(91



98) Vocabulary

A What's in a Name

- 1 A polygon is a flat shape with n straight sides.
- Write down the special name given to a polygon for different values of n.

n = 5	
n = 6	
n = 7	
n = 8	
n = 9	
n = 10	
Describe th	ne special features of a regular polygon.

b) Describe the special features of a regular polygon.

2

acute,	obtuse,	reflex,	right,
isosceles,	scalene	, eq	uilateral

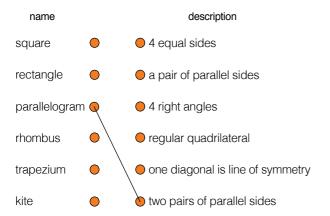
Use names from the box to complete these sentences :

a) A triangle with all its angles under 90° is called

..... angled.

- b) A triangle can not have a angle.
- c) A triangle with all its sides the same length is called

- d) A triangle with two angles the same size is called
- 3 Match the names of quadrilaterals with the best description.

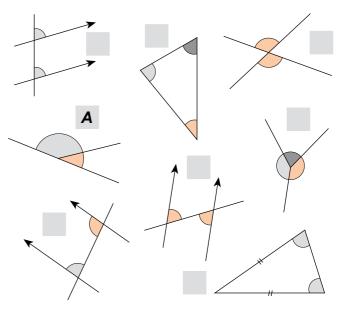


B Angle Rules

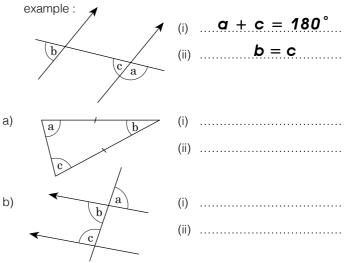
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Rules			
А	Adjacent angles on a straight line add to 180°.		
В	Angles around a point add to 360°.		
С	Vertically opposite angles are equal.		
D	Corresponding angles on parallel lines are equal.		
Е	Alternating angles on parallel lines are equal.		
F	Cointerior angles on parallel lines add to 180°.		
G	Angles inside a triangle add to 180°.		
Н	Base angles in an isosceles triangle are equal.		

The above rules are illustrated with 8 diagrams. Find the correct illustration for each rule and write its letter in the box.



2 For each diagram write two true equations using some or all the labelled angles.



Trigonometry

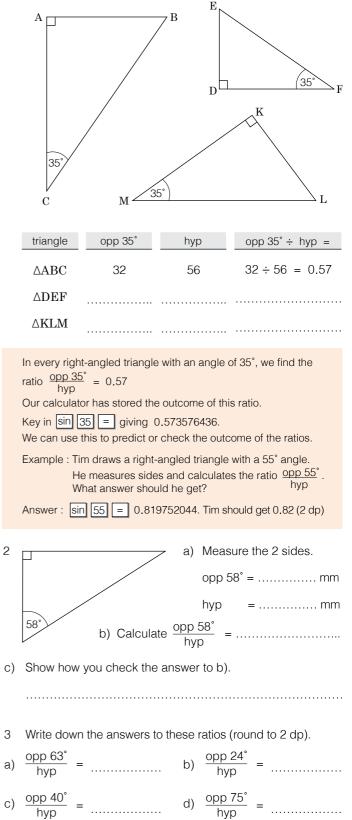
Labelling and Measuring 1



A Labelling 1 In a right-angled triangle the side facing the right angle is called the hypotenuse. 40° If we know the size of another angle, opposite then we can label all sides of the A right-angled triangle (see diagram). Adjacent means lying next to. adjacent 40° The label adjacent 40° is a little misleading because the hypotenuse lies next to the 40° angle also. It is therefore recommended to identify the hypotenuse first. Label the sides of these triangles, start with the hypotenuse. 1 You may use short notations like hyp, opp, adj. 35 b) a) С 35° triangle ∆ABC C) d) $\triangle DEF$ $\triangle KLM$ 45 hvp Use a ruler to measure 2 of the sides. 2 a) ์ 65° mm. hyp 2 opp $65^{\circ} =$ mm. П opp 75° = mm. b) adj 75° = mm. 58 C) C) hyp =..... mm. 3 adj 22° mm. opp 63° a) hvp $\frac{\text{opp 40}^\circ}{\text{hyp}}$ 22

B Opposite and Hypotenuse

In each of these triangles measure the hypotenuse and the side opposite 35° to the nearest millimetre. Fill in the table.



Organising Data

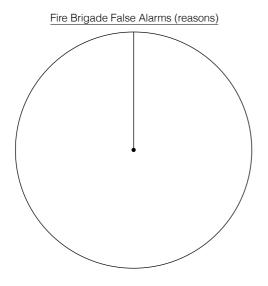
A The Fire Brigade

Pie charts and **strip graphs** are used to graph category data. In these graphs a whole circle or rectangle is cut into fractions. This emphasises the proportion of each relative to the others.

1 Last year the district fire brigade was called out to 144 false alarms. The table shows the reasons.

reason	f	fraction	angle of sector
good intent	45	<u>45</u> <u>144</u>	$\frac{45}{144} \times 360^{\circ} = 112.5^{\circ}$
defective device	32		
malicious	24		
accidental	18		
other	25		
total	144	1	360°

- a) Complete the angle calculations for a pie chart.
- b) Complete the pie chart.



- 2 Now draw a strip graph, 80 mm long, of the false alarms.
- a) The fraction for 'good intent' is $\frac{45}{144}$. How long will the section in the strip graph be?
- b) Draw the strip graph.

Fire Brigade False Alarms (reasons)

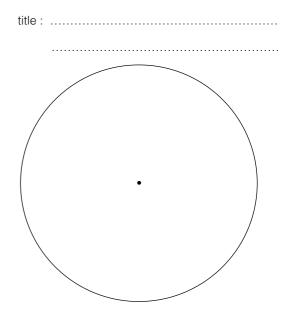
Car Sales

1 This **two way table** shows the number of used cars sold by Urban Motors last year. The cars are listed by size and origin.

Tables and Graphs 3

origin	small	medium	large	Total
Asia	53	74	15	142
Australia	6	10	31	47
Europe	24	26	4	54
Totals	83	110	50	243

- a) You are asked to draw a pie graph to display the origin of the used cars sold by Urban Motors last year.
 - i) First show how you work out the angle of the sector with origin *Asia*.
 - ii) Complete the pie graph.



- b) You are asked to draw a strip graph (75 mm long) to display the size of the used cars sold by Urban Motors last year.
 - i) First show how you work out the length of the section with size *small*.
 - ii) Complete the strip graph.

Size of Used Cars Sold by Urban Motors

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Probability and Proportion

Chapter 11 Estimating Probability

Past Experience

To work out the probability of an event, we can look at **patterns** that have happened in the past.

Example :

A shop sells iPad Airs in different colours.	colour of iPad Air	number	
The table shows how many of each colour were	IPad Air	sold	
sold last month. Calculate the probability that the	grey	27	
next person who buys an iPad Air chooses gold.	gold	19	
her person who buys arm au Air chooses goid.	0		
Working : In the past 19 out of 88 people chose	silver	42	
o	total	88	
gold iPad Air ; $\frac{19}{88} = 22\%$	totai	00	
Answer : The probability that the next buyer chooses a gold iPad Air is 22%.			
Notation : $P(gold) = 22\%$			

1 This table shows the number of schools there were in New Zealand in the year 2015.

type of school	number of schools
Primary & Intermediate	1963
Composite Schools	169
Secondary Schools	344
Special Schools	38
total	

Source : Ministry of Education

One school in New Zealand is chosen at random. Calculate these probabilities (in %).

- a) P(Secondary School)
- =%

.....

b) P(not Secondary School) =%

2 This table shows the result of a survey on how long unemployed people had been on the Community Wage Scheme.

> One unemployed person on the Community Wage Scheme is chosen at

random. Calculate the

probability that . . .

time on the scheme	N ^{o.} people
up to 1 month	15
1 - 3 months	40
3 - 6 months	28
6 - 12 months	22
over 1 year	25

a) the person has been unemployed for over a year.

b) the person has been unemployed for up to 6 months.

B Multivariate Tables

Example :					
This table shows the type of books on a bookshelf.	type	hard back	paper back	total	
a) How many hardback	non-fiction	6	4	10	
fiction books are there?	fiction	2	20	22	
b) One book is taken off the shelf at random, calculate	total	8	24	32	
i) P(paperback non-fiction) ii) P(fiction)	1			
c) One non-fiction book is chosen at random, calculate the probability that it is a hardback.					
Answers : a) 2 b) i) $\frac{4}{32}$ = 13% ii) $\frac{22}{32}$ = 69% c) $\frac{6}{10}$ = 60%					

1 In December students of Year 9 made their subject choices for Year 10. The table shows the choices made for option A.

Yr 10, Option A	number of boys	number of girls	total
Accounting	42	35	
Agriculture	10	4	
Art	36	53	
total			180

- a) One student in this group of 180 is chosen at random.
 Calculate the probability that the student is . . .
 - i) a girl who chose Accounting
 - ii) a boy who did not choose Art
- b) One Agriculture student is chosen at random. Calculate the

probability this student is a girl.

2a) The table shows numbers of overseas visitors to NZ and their reasons for visiting us. Complete the table.

country of residency	business	holiday	total
Australia	148 000	444 000	
UK		175 000	
USA	41 000	154 000	
Other	172 000		850 000
total	396 000		

Source : Statistics NZ

- b) One overseas visitor is chosen at random.
 Calculate the probability that the visitor is a UK resident.
- c) One overseas holiday maker is chosen at random.
 Calculate the probability that the visitor is from Australia.