Integers, Powers and Roots 3 — 6

A Negative Powers

1 Complete these tables and use them to understand the meaning of negative powers.

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
16	***8***					

÷ 2

3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}
81	***27***				$\frac{1}{3}$	

÷ 3

2 Write these negative powers as a fraction and a decimal.

power	fraction	decimal
2^{-1}	$\frac{1}{2}$	0.5
2^{-2}		
2^{-3}		
5^{-1}		
5^{-2}		
10^{-1}		
10^{-3}		

3 Find.

a) 5^0 .... b) 10^0 ....

4a) If n is any whole number, what is *"n to the power of zero?"*

....

b) Write n^{-3} as a fraction. ....

5 Use the fact that $2^{12} = 4096$ to calculate 2^{-11}.

....

6 Calculate.

a) 2^{-5} ....

b) 4^{-4} ....

B Extension Exercise

We know : $a^2 = a \times a$ and $a^{-2} = \frac{1}{a \times a}$,
but what is meant with $a^{\frac{1}{2}}$, $a^{\frac{1}{3}}$, $a^{\frac{1}{4}}$, etc?

Answer : $a^{\frac{1}{2}} = \sqrt{a}$, $a^{\frac{1}{3}} = \sqrt[3]{a}$, $a^{\frac{1}{4}} = \sqrt[4]{a}$

Mentally calculating fractional powers is done the same way as calculating roots : use *guess and check*.

For example : $81^{\frac{1}{2}} = \sqrt{81} = 9$, because $9^2 = 81$
$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$, because $4^3 = 64$
$625^{\frac{1}{4}} = \sqrt[4]{625} = 5$, because $5^4 = 625$

1 Calculate mentally.

a) $4^{\frac{1}{2}}$ ....

b) $1000^{\frac{1}{3}}$ ....

c) $16^{\frac{1}{4}}$ ....

d) $1^{\frac{1}{6}}$ ....

e) $32^{\frac{1}{5}}$ ....

f) $(\frac{1}{4})^{\frac{1}{2}}$ ....

The [^] key on your calculator can be used to calculate any powers, positive, negative or fractional. For negative powers use the negative key [(–)], for fractional powers use the [▤] key, remember to put brackets around the power.

For example : $125^{\frac{2}{3}}$ is keyed in as . . .

[125] [^] [(] [2] [▤] [3] [)] [EXE],
Answer : 25

2 Check your answers to Q1 with a calculator.

3 With your calculator work out . . .

a) $16^{\frac{3}{4}}$ .... b) $9^{\frac{3}{2}}$ ....

4 Use your answers to Q3 above to explain the meaning of . . .

a) $a^{\frac{3}{4}}$ ....

b) $a^{\frac{3}{2}}$ ....

5 Calculate with or without your calculator.

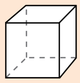
a) $27^{\frac{2}{3}}$ ....

b) $64^{\frac{1}{6}}$ ....

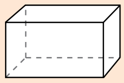
c) $8^{\frac{4}{3}}$ ....

A Faces

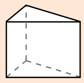
A cube has 6 square faces.
All its faces are congruent.



A cuboid has 6 faces, they are rectangles or squares. Opposite faces on a cuboid are congruent.



A triangular prism has 5 faces. Two are congruent triangles, the other 3 are rectangles.

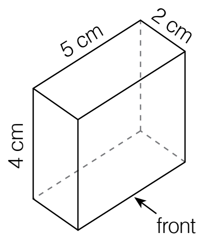


(See Page 21 for more information on Congruence.)

1 What does the word congruent mean? ....

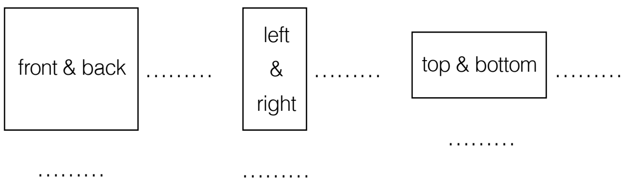
....

2a) What is the name of this solid?

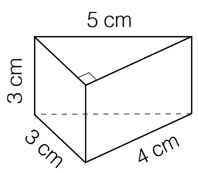


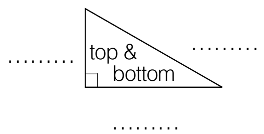
....

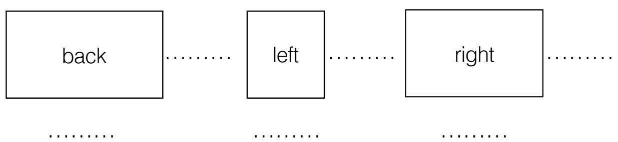
b) Here are sketches of its faces. Write the dimensions (lengths and widths) on the sketches.



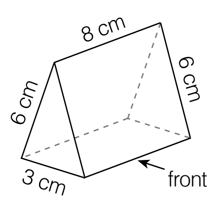
3 Write the dimensions on the faces of this prism.







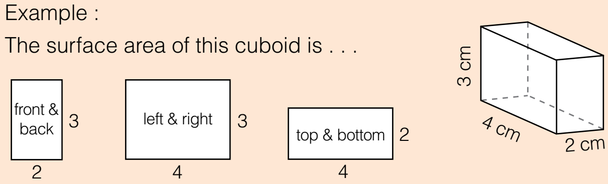
4 Sketch the faces of this triangular prism. Include the dimensions.



B Surface Area

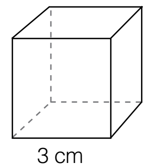
The surface area of a solid is the total area of all its faces.

Example :
The surface area of this cuboid is . . .



$6 + 6 + 12 + 12 + 8 + 8 = 52\ cm^2$

1 Calculate the surface area of a cube with edges of 3 cm.



....

....

2 Calculate the surface area of the cuboid in Exercise A 2.

....

....

3 Calculate the surface area of the prism in Exercise A 3.

....

....

4 Can you calculate the surface area of the prism in Exercise A 4? Explain.

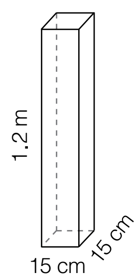
....

....

....

....

5 Calculate the surface area of this pole. (Hint : work in cm)



....

....

....

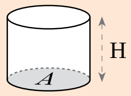
....

....

....

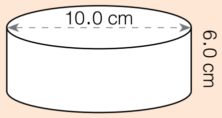
A Volume of Cylinders

The volume of a cylinder is also found with the formula $V = A \times H$, where A is the area of the circular base, H the height of the cylinder.



Example :

Calculate the volume of this cylinder.



Working :

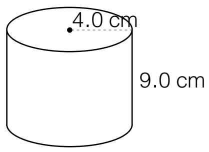
$A = \pi \times r^2 = \pi \times 5^2$; $H = 6$

$V = \pi \times 5^2 \times 6 = 470 \text{ cm}^3$ (2 sf)

Note : The value of A is not worked out. The value of V is calculated with unrounded numbers and sensible rounding is done at the very end.

1 Calculate the volumes of these cylinders.

a)



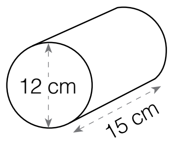
A = ...

H = ...

V = ...

....

b)



A = ...

H = ...

V = ...

....

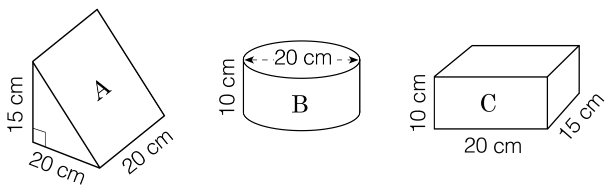
2a) Calculate the volume of a coffee mug with a diameter of 7 cm and a height of 8.5 cm (measured *inside* the mug).

....

....

b) Convert your answer to millilitres. ....

3 Which of these solids has the largest volume?



....

....

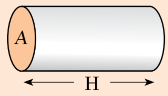
....

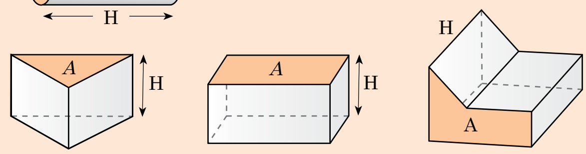
....

....

B Same Cross-section

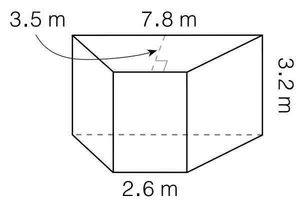
The volume of cubes, cuboids, prisms and cylinders can all be found with one formula $V = A \times H$, where A is the area of the cross section, and H is the height, measured perpendicular to the cross section.





1 Calculate the volume of these solids.

a)



....

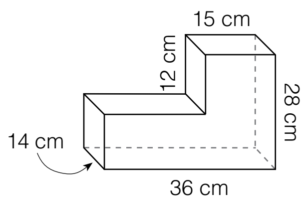
....

....

....

....

b)



....

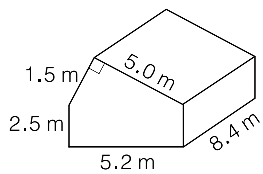
....

....

....

....

c)



....

....

....

....

....

2 This kiddy pool is filled with water 25 cm deep.

a) The area of the base is 1.8 m^2. What is the area in cm^2?

....

....

b) How many litres of water are in the kiddy pool?

....

....

A Calculating the Scale Factor

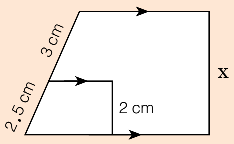
To find the scale factor for the sides, you first find matching sides.

The scale factor is found by : $k = \dfrac{\text{length long side}}{\text{length short side}}$

Example : The diagram shows two similar trapezia.

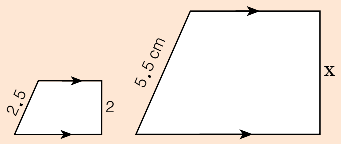
a) Calculate the scale factor.

b) Calculate the length of x.



Working :

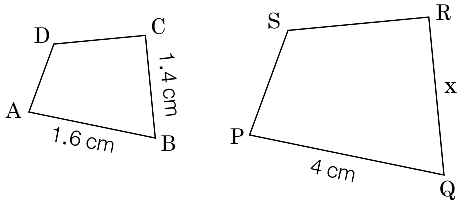
Lift the small shape out of the large shape.



a) $k = \dfrac{5.5}{2.5} = 2.2$

b) $x = 2.2 \times 2 = 4.4$ cm

1 These two similar shapes are not drawn to scale.



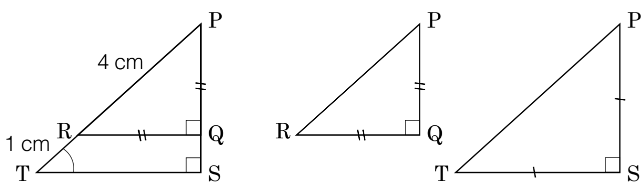
a) Calculate the scale factor (k). ....

b) Calculate the length of side $\overline{QR}$ (x).

....

....

2



Isosceles triangle PQR is similar to triangle PST.

a) Calculate ∠STP.

....

....

b) Calculate the scale factor.

....

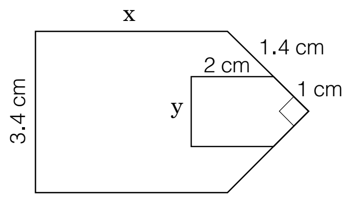
c) Side $\overline{RQ}$ = 2.8 cm. How long is side $\overline{TS}$?

....

....

B Big and Bigger

1 A small pentagon has been enlarged as shown.



a) Draw the two pentagons separately.

b) Calculate the scalefactor, k.

....

c) Calculate lengths x and y.

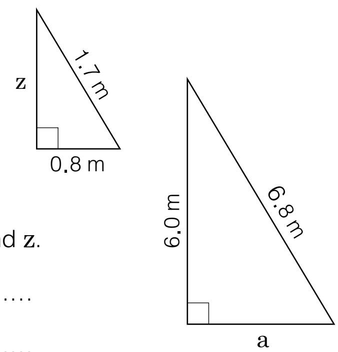
....

....

....

....

2 These triangles are similar but not drawn to scale.



a) Calculate the scalefactor for the sides, k.

....

b) Calculate the lengths of a and z.

....

....

....

....

c) Calculate the areas of the triangles.

Area small Δ = ...

....

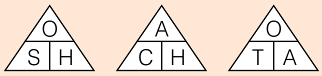
Area large Δ = ...

....

d) Complete this sentence : *'The area of the large triangle is* .... *times the area of the smaller triangle'.*

A Four Steps to Calculate a Side

If in a right-angled triangle you know the size of one more angle and the length of one side, then you can use the ratio triangles to calculate any of the other two sides.

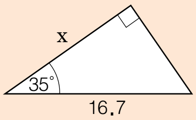


The calculation has 4 steps :

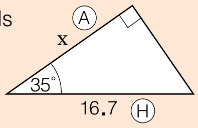
1) In the right-angled triangle label two of the sides with **H** (hypotenuse) or **O** (opposite) or **A** (adjacent).
Only label the side you know and the side you want to know.
2) Choose the relevant ratio triangle : SOH, CAH, or TOA.
3) Substitute known values into the ratio triangle.
4) Calculate the length of the side using your calculator and round sensibly.

Example : Calculate x.

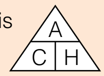
Working :



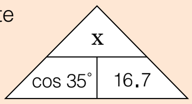
1) Labels



2) With labels A and H the choice is



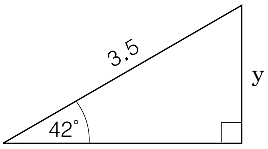
3) Substitute values.



4) Calculate and round.

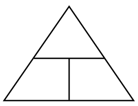
x = cos 35° x 16.7
= 13.7 (3 sf)

1 We will use the 4 step method to calculate the length of side y.

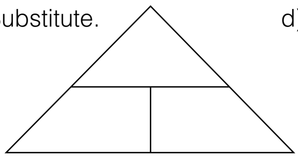


a) Label sides 'y' and '3.5'.

b) Choose SOH, CAH or TOA.



c) Substitute.



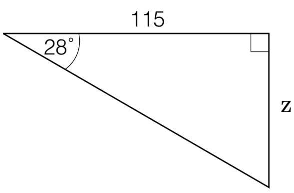
d) Calculate and round y.

y = ...

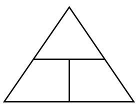
....

2 Calculate the length of side z in four steps.

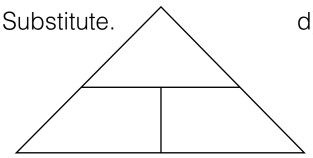
a) Label sides 'z' and '115'.



b) Choose SOH, CAH or TOA.



c) Substitute.



d) Calculate and round z.

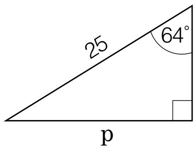
z = ...

....

B On Your Own

1 Calculate the labelled sides, round sensibly.

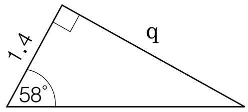
a)



....

....

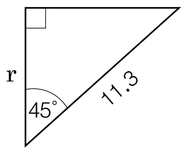
b)



....

....

c)

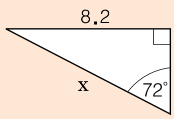


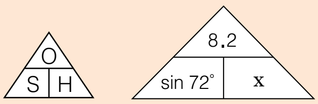
....

....

Example : Calculate side x.

Working : x has label H
8.2 has label O

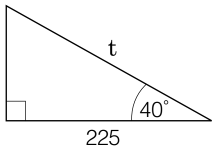




$x = \dfrac{8.2}{\sin 72^\circ} = 8.6$ (2 sf)

2 Calculate the labelled sides, round sensibly.

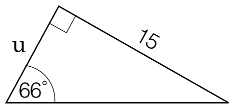
a)



....

....

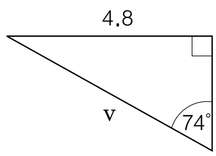
b)



....

....

c)



....

....

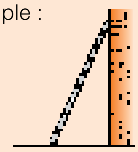
37 Applications of Trigonometry 1

A Labels and Units

In a word problem the side to be calculated is not usually marked with x, you have to do that yourself.

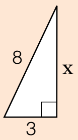
The unit of measurement is important. Check that all given measurements have the same unit, the answer will get that same unit. While doing the calculations however, you ignore the unit and work with numbers only.

Example :



A house painter has a ladder which extends to 8 m.
The foot of the ladder is placed 3m from the building.
How high up the building does the ladder reach?

Working :



$$x^2 + 3^2 = 8^2$$
$$x^2 = 8^2 - 3^2$$
$$x = \sqrt{8^2 - 3^2}$$
$$x = 7.4 \text{ (1dp)}$$

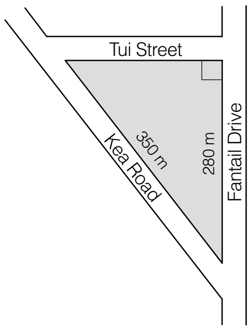
← ignore the unit

write the unit

Answer : The ladder reaches 7.4 m (2 sf) up the building.

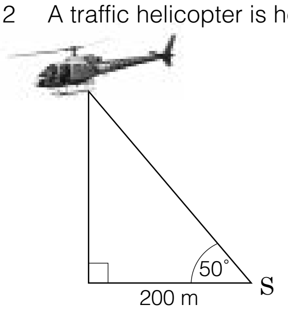
1 This map shows a park surrounded by three roads.

Calculate the length of Tui street.



Answer : ....

2 A traffic helicopter is hovering above a roundabout.



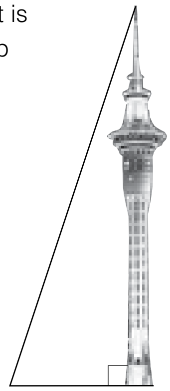
Sasha is 200 m away from the roundabout. When looking up at the helicopter, the angle above the horizontal is 50°.

Calculate the height of the helicopter above the ground.

Answer : ....

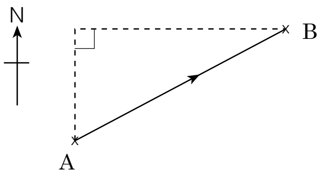
B Place the Measurements

1 The Auckland Skytower is 328 m high. What is the angle between the horizontal and the top of the Skytower at a point 90 m from the entrance?



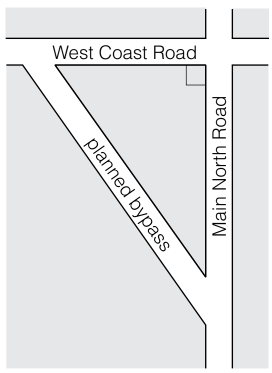
Answer : ....

2 Airport B is 150 km North and 220 km East of airport A.
A plane takes off at A and lands at B.
What distance did it fly?



Answer : ....

3



The council is planning a bypass to relieve the busy intersection between West Coast Road and Main North Road. The bypass will make an angle of 35° with Main North Road and it will join West Coast Rd 2.5 km from the intersection.

Calculate the length of the bypass.

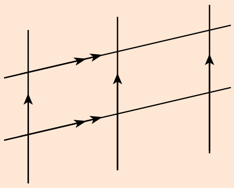
Answer : ....

45 Basic Angle Rules 2

A Parallel Lines

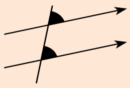
If in a diagram a set of lines is marked with the same type of arrows then we know that these lines are **parallel**.

This diagram shows 3 parallel lines going up. They are crossed by another pair of parallel lines.

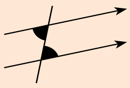


A line crossing parallel lines is called a **transversal**. There are 3 rules about angles formed by a transversal and parallel lines.

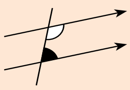
Rule : Corresponding angles on parallel lines are equal.
(corr ∠s // lines are =)



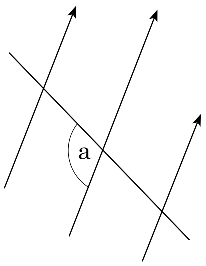
Rule : Alternating angles on parallel lines are equal.
(alt ∠s // lines are =)



Rule : Co-interior angles on parallel lines add to 180°.
(co-int ∠s // lines add to 180°)



1

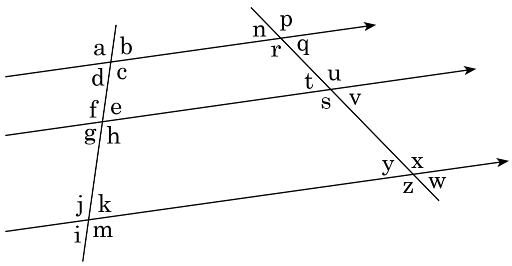


a) Colour red the two angles that are corresponding to angle a.

b) Colour blue the angle that is alternating with angle a.

c) Colour green the angle that is co-interior with angle a.

2



a) Why is $c = h$? ....

....

b) Why is $f + g = 180°$? ....

....

c) Why is $t = q$? ....

....

d) Why is $s + y = 180°$? ....

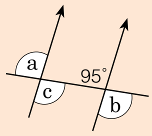
....

e) Which of these is true? Circle the correct one.

A $c + r = 180°$ B $c + t = 180°$ C $c + k = 180°$

B Giving Reasons

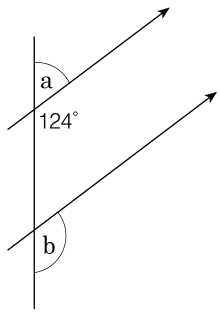
Example :



Calculate angles a, b and c.
Give reasons for your answers.

Answer : a = 95° (corr ∠s // lines are =)
b = 95° (vert opp ∠s are =)
c = 95° (alt ∠s // lines are =)

1



Calculate angles a and b.
Give reasons for your answer.

....

....

....

....

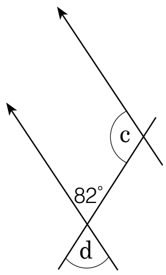
2 Calculate angles c and d.
Give reasons for your answer.

....

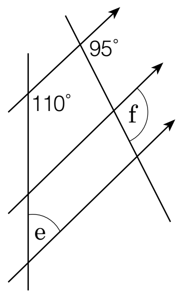
....

....

....



3



Calculate angles e and f.
Give reasons for your answer.

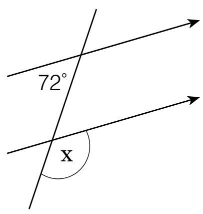
....

....

....

....

4



Calculate angle x.

There is no rule connecting x with the marked angle of 72°.

In the diagram mark another angle which could be used as a stepping stone. Label this angle with w and work out its size.

w = ... reason ....

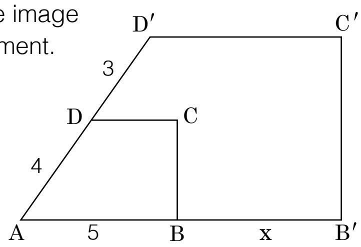
....

x = ... reason ....

....

A An Enlargement

1 Trapezium $AB'C'D'$ is the image of $ABCD$ after an enlargement.
$\overline{AB}$ = 5 cm,
$\overline{AD}$ = 4 cm,
$\overline{DD}'$= 3 cm.



a) Calculate the length of $\overline{BB}'$, marked **x**.

....

....

....

....

....

....

b) Also given : $\angle BAD = 68^\circ$, $\angle ABC = 90^\circ$.
Calculate the areas of the two trapezia.
Explain your reasoning.

....

....

....

....

....

....

....

....

....

....

....

....

....

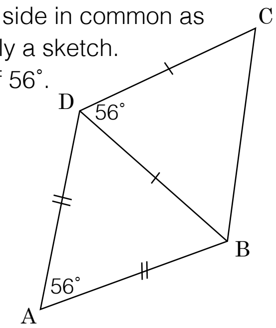
....

....

....

B Two Triangles

1 Two isosceles triangles have one side in common as shown in the diagram which is only a sketch.
Both triangles have a top angle of 56˚.



a) Are the two triangles congruent?
Justify your answer.

....

....

....

....

....

....

....

....

....

....

b) What is the best name for quadrilateral ABCD?
Justify your answer.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

Simplifying an expression means writing it in the shortest possible form, without changing the value of the expression.

For example, we can simplify the expression $3a \times 5a$ to $15a^2$.

To check our answer we could substitute $a = 2$, then the value of $3a \times 5a = 3 \times 2 \times 5 \times 2 = 60$, and also $15a^2 = 15 \times 4 = 60$. The value is unchanged.

A First Long, Then Short

It is often a good idea to write a multiplication in long form first, then combine terms to write the shortest possible expression.

Examples : $3a \times 5a = 3 \times a \times 5 \times a = 15a^2$

$2a^2 \times {}^-4ab = 2 \times a \times a \times {}^-4 \times a \times b = {}^-8a^3b$

1 Write these in long form, then simplify.

a) $4y \times 3$ ....

b) $8 \times {}^-2w$ ....

c) $2y \times 3y$ ....

d) $3w \times 6y$ ....

e) $5z^2 \times 3z$ ....

f) $y^4 \times y^3$ ....

g) $w^3 \times w$ ....

h) ${}^-3y^2 \times {}^-2yz$ ....

i) $2y^2z \times 5z^2$ ....

2 Simplify.

a) $2a \times 2b \times 2a$ ....

b) ${}^-2a \times {}^-4a \times a$ ....

c) $ab^2 \times {}^-2a \times 3b$ ....

3 Simplify these without writing the long form first.

a) $a^2 \times a^3$ .... b) $a^2 \times ab$ ....

c) $b \times 5b$ .... d) $b \times b^4$ ....

e) $2a^2 \times 4ab$ .... f) $3ab \times a^3$ ....

g) $a^2 \times a^3 \times a$ ....

h) $a^6 \times a^5 \times b^2 \times b$ ....

i) $3a \times 3a \times 3a \times 3a$ ....

j) $5 \times {}^-2a^2 \times {}^-2a^2 \times {}^-2a^2$ ....

B Look, No Brackets!

The square of a term equals the term times itself.

Therefore : $(3a)^2 = 3a \times 3a = 9a^2$

and $2(3a)^2 = 2 \times 3a \times 3a = 18a^2$

1 Write these expressions without brackets.

a) $(4a)^2$ ....

b) $({}^-2a)^3$ ....

c) $(a^3)^2$ ....

d) $(b^2)^3$ ....

e) $(3a^2)^2$ ....

f) $(ab)^4$ ....

g) $(a^2b)^2$ ....

h) $2(a^2b)^2$ ....

....

i) $3a(2a^2)^2$ ....

....

2a) Circle your answer to this multichoice question.
$(2a^2)^5$ simplifies to :

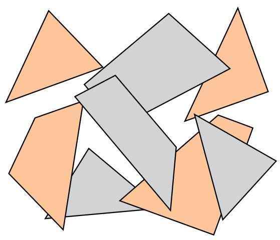
A $10a^{10}$ B $32a^{10}$ C $10a^7$ D $32a^7$

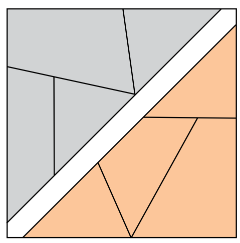
b) Check your answer by substituting $a = 2$.

....

....

....





A More Than Once

If the variable occurs more than once, we must first **simplify** the equation and then **solve** as usual.

Example : Simplify and solve.

a) $2x - 8x = 12$ b) $2a - 4 + 3a = 2$

Working :

a)
$2x - 8x = 12$
$^{-}6x = 12$ $(\div {}^{-}6)$
$x = {}^{-}2$

b)
$2a - 4 + 3a = 2$
$5a - 4 = 2$ $(+ 4)$
$5a = 6$ $(\div 5)$
$a = 1.2$

1 Simplify and solve.

a) $5x + 3x = 24$
b) $7x - 5x = 10$
c) $2a + a = {}^{-}9$
d) $8a - a = 28$
e) $3y - 7y = 20$
f) $y - 6y = {}^{-}30$

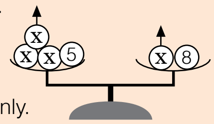
2 Solve.

a) $5x + 4x + 3 = 30$
b) $x + x - 8 = 4$
c) $6a + 8 + 3a = {}^{-}10$
d) $a - 1 + 2a = 26$
e) $3y + 6 - 5y = 2$
f) $^{-}8y + 3 + 5y = 6$

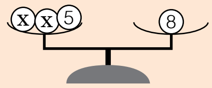
B Balancing Act

If the variable occurs on both sides of the equation, then we can use the **balance method** to solve it. A balance stays stable if we take away the same amount from each side.

Example : Solve $3x + 5 = x + 8$



Take away one x from each side. We get a new equation with the variable on one side only. Then solve as usual.



$2x + 5 = 8$ $(- 5)$
$2x = 3$ $(\div 2)$
$x = 1.5$

1 Solve.

a) $3x + 2 = x + 12$
b) $5x + 1 = x + 9$
c) $6x - 1 = 3x + 2$
d) $5x - 8 = 2x + 1$

More examples :

a) $2x - 1 = 5x + 2$

take away $5x$ from each side

$^{-}3x - 1 = 2$ $(+ 1)$
$^{-}3x = 3$ $(\div {}^{-}3)$
$x = {}^{-}1$

b) $x + 4 = 10 - 2x$

add on $2x$ to each side

$3x + 4 = 10$ $(- 4)$
$3x = 6$ $(\div 3)$
$x = 2$

2 Solve.

a) $2x - 1 = 4x + 7$
b) $x - 5 = 7 + 5x$
c) $5x + 2 = 8 - x$
d) $x - 1 = 5 - 3x$
e) $2x - 4 = 3x + 1$
f) $3x + 4 = 11 - 2x$

A Trinomials

An expression of the form $a^2 + 5a + 6$ is called a **trinomial**. It has 3 parts. Factorising a trinomial is the opposite of expanding double brackets.

Example : Factorise $a^2 + 5a + 6$

Working :

(5a)

a^2	
	6

Place the expression in this box. Problem : the box has 4 parts, the expression has 3. But we know the two blank parts add to 5a. Now find the numbers on the outside.

a^2 is made by $a \times a$ and 6 can be made by 1 x 6 or by 2 x 3

(7a) WRONG

	a	6
a	a^2	6a
1	1a	6

(5a) RIGHT

	a	3
a	a^2	3a
2	2a	6

Answer : $a^2 + 5a + 6 = (a + 2)(a + 3)$

Note that $(a + 2)(a + 3)$ is the same as $(a + 3)(a + 2)$

1 Factorise these trinomials.

a) $x^2 + 7x + 10$

= (……………) (……………)

(7x)

x^2	
	10

b) $a^2 + 10a + 24$

= (……………) (……………)

(10a)

a^2	
	24

c) $y^2 + 2y + 1$

……………………………………

d) $p^2 + 9p + 8$

……………………………………

e) $a^2 + 8a + 12$

……………………………………

f) $x^2 + 8x + 16$

……………………………………

B More Trinomials

When a trinomial has subtraction in it, it can be a puzzle to find fitting numbers.

Example : Factorise $y^2 + 3y - 10$

(3y)

y^2	
	⁻10

y^2 is made by $y \times y$

⁻10 can be made by . . .

⁻1 x 10   1 x ⁻10   ⁻2 x 5   2 x ⁻5

⁻2 x 5 works because ⁻2 x 5 = ⁻10 *and* ⁻2 + 5 = 3.

Answer : $y^2 + 3y - 10 = (y - 2)(y + 5)$

1 We will factorise $p^2 - 7p + 12$

(⁻7p)

p^2	
	12

+12 can be made by . . .

1 x 12   ⁻1 x ⁻12   2 x 6

⁻2 x ⁻6   3 x 4   ⁻3 x ⁻4

a) Try it out. Which one works? …………………

b) Complete : $p^2 - 7p + 12$ = (………………) (………………)

2 We will factorise $a^2 + 5a - 24$.

(5a)

a^2	
	⁻24

⁻24 can be made by . . .

⁻1 x 24   1 x ⁻24   ⁻2 x 12

2 x ⁻12   ⁻3 x 8   3 x ⁻8

4 x ⁻6   ⁻4 x 6

a) Try it out. Which one works? …………………

b) Complete : $a^2 + 5a - 24$ = (………………) (………………)

3 Factorise these.

a) $x^2 + 2x - 35$

………………………………………………

b) $y^2 - 8y + 7$

………………………………………………

c) $p^2 - 2p - 15$

………………………………………………

91 Solving Quadratic Equations 4

A Faster!

A quadratic equation of the form $(x + b)^2 = c$ can be solved in a faster way.

Examples :

a) $x^2 = 16$
$x = 4$ or $x = {}^-4$

b) $(x + 3)^2 = 25$
$x + 3 = 5$ or $x + 3 = {}^-5$
$x = 2$ or $x = {}^-8$

c) $(x - 4)^2 = 6$
$x - 4 = \sqrt{6}$ or $x - 4 = {}^-\sqrt{6}$
$x = \sqrt{6} + 4$ or $x = {}^-\sqrt{6} + 4$

1 Solve.

a) $x^2 = 36$ $x =$ .... or $x =$ ....

b) $x^2 = 20$ $x =$ .... or $x =$ ....

2 Solve.

a) $(x + 2)^2 = 16$ ....

....

b) $(x - 1)^2 = 100$ ....

....

c) $(2x + 3)^2 = 64$ ....

....

....

d) $(x + 3)^2 = 50$ ....

....

....

e) $(x - 2)^2 = 28$ ....

....

....

3 Rearrange and solve.

a) $2(a + 1)^2 = 8$ ....

....

....

b) $(2a - 3)^2 - 6 = 10$ ....

....

....

B A Quadratic Pattern

1 We will investigate a pattern of number sentences.

a) Carefully check the correctness of the number sentences in this pattern.

$\frac{36 - 1}{6 - 1} = 6 + 1$	✓	$\frac{35}{5} = 7$
$\frac{49 - 4}{7 - 2} = 7 + 2$		
$\frac{64 - 16}{8 - 4} = 8 + 4$		
$\frac{36 - 1}{9 - 1} = 4 + 1$		
$\frac{64 - 4}{32 - 2} = 2 + 2$		

b) Complete these sentences correctly.

i) $\frac{16 - 9}{\text{.........}}$ = ... ii) $\frac{25 - 49}{\text{.........}}$ = ...

c) Generalise the pattern for any set of numbers a and b.

$\frac{a^2 - b^2}{\text{.........}}$ = ...

d) Show that the pattern is no surprise.

....

....

....

2 $x^2 + nx - 12 = 18$ is a quadratic equation where n is a whole number. The difference between the solutions to the equation is 13. Find the value of n.

....

....

....

....

....

....

....

....

....

....

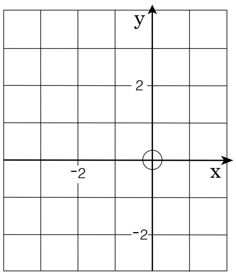
....

....

A Gradient and Y-intercept

1 Graph the line and write down the gradient and the y-intercept.

a) $y = x + 3$



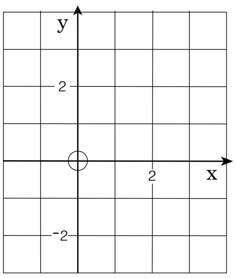
....

....

....

grad = ... y-int = ...

b) $y = {}^{-}2x + 1$



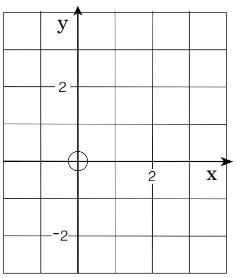
....

....

....

grad = ... y-int = ...

c) $y = \frac{2}{3}x - 2$



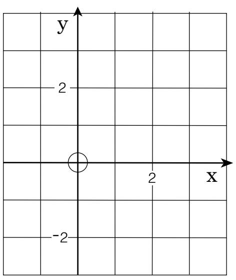
....

....

....

grad = ... y-int = ...

d) $y = 1 - 3x$



....

....

....

grad = ... y-int = ...

Remember : If a rule is written in the form $y = mx + c$, then m is the gradient, c is the y-intercept.

2 Without drawing the graph, write down the gradient (m) and y-intercept (c) for these lines.

a) $y = \frac{1}{2}x + 5$ m = ... c = ...

b) $y = 0.3x - 1$ m = ... c = ...

c) $y = 4 - 2x$ m = ... c = ...

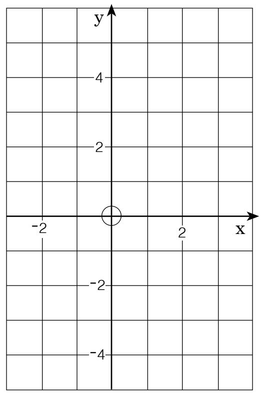
d) $y = \frac{4}{5} + \frac{1}{5}x$ m = ... c = ...

e) $y = {}^{-}2$ m = ... c = ...

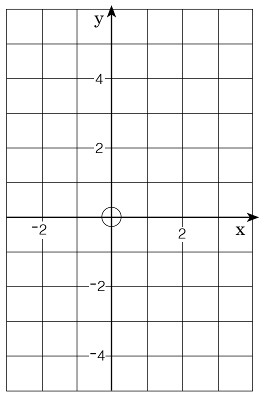
B Draw Using m and c

1 Draw line p which has gradient ${}^{-}2$ and y-intercept (0, 3).

Hint : start by plotting the y-intercept, then use the gradient to plot another point. Join the points.



2 Draw line q which has gradient $\frac{2}{3}$ and y-intercept (0, ${}^{-}1$).

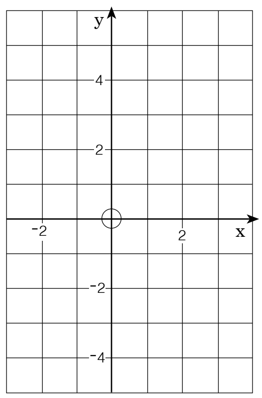


3 Line r has equation $y = \frac{3}{4}x - 2$.

a) Fill in :

m = ... c = ...

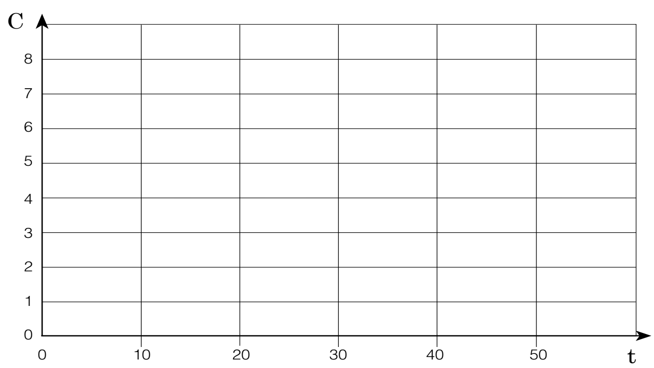
b) Draw line r.



4 Draw the graph of $C = \frac{-3}{20}t + 6$

....

....



A Equation $y = \pm(x - p)(x - q)$

The parabolas in this exercise are not drawn to scale, but it is known their equation is of the form $y = \pm(x - p)(x - q)$.
For each parabola you are asked to . . .

a) Write the equation,
b) Work out the y-intercept,
c) Work out the coordinates of the vertex.

1a) Equation :

....

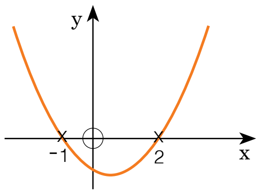
b) y-intercept :

....

c) vertex : ....

....

....



2a) Equation :

....

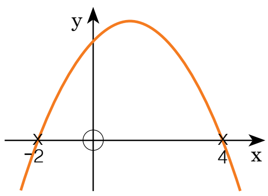
b) y-intercept :

....

c) vertex : ....

....

....



3a) Equation :

....

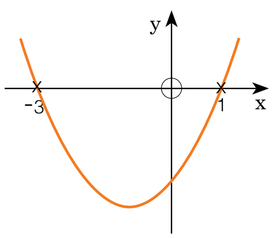
b) y-intercept :

....

c) vertex : ....

....

....



4a) Equation :

....

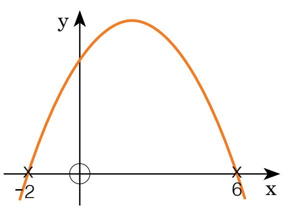
b) y-intercept :

....

c) vertex : ....

....

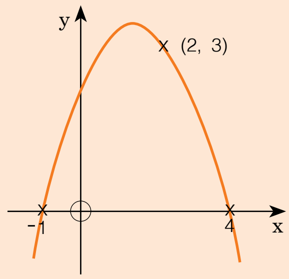
....



B Equation $y = a(x - p)(x - q)$

Example :
Write an equation for this parabola.

Working : Using the x-intercepts
$y = a(x + 1)(x - 4)$
The value of a can be calculated as follows :
since (2, 3) is on the parabola, we substitute $x = 2$ and $y = 3$ in the equation :
$y = a(x + 1)(x - 4)$
$3 = a(2 + 1)(2 - 4)$
$3 = a \times 3 \times {}^-2$ then $3 = {}^-6a$ so $a = {}^-\frac{1}{2}$.
Equation : $y = {}^-\frac{1}{2}(x + 1)(x - 4)$



1 Write an equation for each graph.

a) ....

....

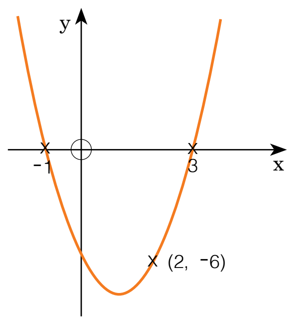
....

....

....

....

....



b) ....

....

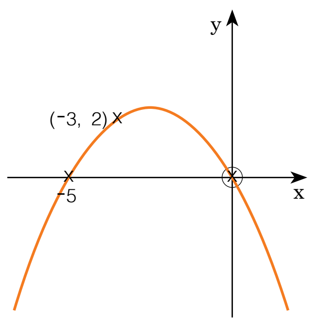
....

....

....

....

....



c) ....

....

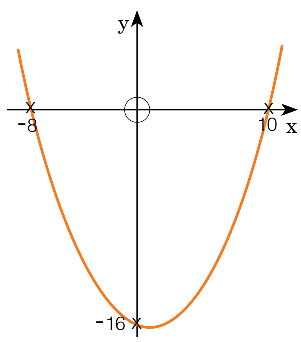
....

....

....

....

....



A Looking for a Perfect Fit

When a relationship is given in a table, you can use a **graphic calculator** or a **spreadsheet** to find an equation of the relationship.

General steps :

1] enter the data in two columns,

2] take steps to display a scatter graph,

3] examine the plot and try out a shape of graph (or trendline) e.g. line, parabola, exponential curve,

4] the calculator/spreadsheet can give you the equation of the line or curve and will also indicate how well this equation fits the given data set (check that $r^2 = 1$),

5] if $r^2 < 1$ then the line/curve is not a perfect fit and another curve should be tried (back to step 3).

Example : Find an equation for the relationship between x and y.

x	1	2	3	4	5	6
y	0.25	0.5	2.25	5.5	10.25	16.5

Working : These are the steps for a *Casio fx 9750* or *9860 GIII*.

Select **STAT** from the main menu. If there are values in the lists, find **DEL-A** (delete all) on the bottom menu and wipe the data.

1] In **List 1** enter the x-values, in **List 2** enter the y-values (press **EXE** after each entry).

2] Choose **GRPH** (**F1**) from the bottom menu.

a) Choose **SET**, then **GPH1** from the bottom menu (we are going to set up **StatGraph1** as a scattergraph, use the **REPLAY** arrow to scroll down).

b) set **Graph Type** to *Scatter* [select **Scat** (**F1**)]

c) set **Xlist** to **List 1** (choose **LIST** (**F1**) and then [1], **EXE**)
set **Ylist** to **List 2** (choose **LIST** (**F1**) and then [2], **EXE**).

d) **frequency** should be set to **1**, choose any marktype.

e) press **EXE** and you see the table again.

3] To display the scatter graph select **GPH1** (**F1**).
Examine the curve - in this case the curve could be *exponential* or a *parabola*.

4] Select **CALC** (**F1**)
We first try an exponential equation. On the bottom menu scroll over and select **EXP** (**F3**), since we wish the formula to be of the form $y = a\,b^x$, select ab^x (**F2**).
The screen gives us the values of a and b, but the most important at this stage is the value of r^2. Since r^2 is less than1, the curve is not a perfect fit. This becomes obvious when you select **DRAW** (**F6**), the curve does not pass through the points.
Press the **EXIT** key.

5] We will now try to fit a parabola. Scroll the bottom menu till you find x^2 (**F4**). The formula will be of the form $y = ax^2 + bx + c$. The screen shows us values of a, b and c, but most importantly it shows $r^2 = 1$, so the equation is a perfect fit.
Solution : $y = 0.75x^2 - 2x + 1.5$
- select **DRAW**(**F6**) to check!

Note : If you want to try to fit a straight line, select X (**F2**) from the bottom menu.

B Practice Makes Perfect

For each table . . .

a) find a linear, quadratic or exponential equation that fits.

b) work out the value of y for $x = 10$.

1

x	0	1	2	3	4	5
y	1	2.5	7	14.5	25	38.5

a) ....

b) ....

2

x	0	1	2	3	4
y	10.35	12.6	14.85	17.1	19.35

a) ....

b) ....

3

x	0	1	2	3	4
y	2.5	5.5	12.1	26.62	58.564

a) ....

b) ....

4

x	2	4	8	12	18	20
y	75	151	315	495	795	903

a) ....

b) ....

5a) Find the best possible fit for this relationship.

x	0	2	3	5	6
y	1000	1210	1331	1610.5	1771.6

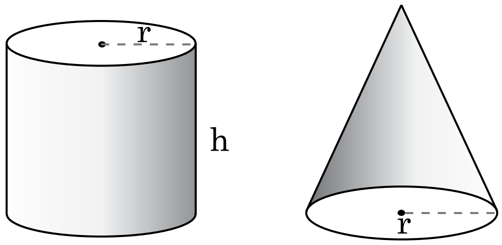
....

....

b) Calculate y for $x = 10$. ....

....

9 The volume of a cylinder is given by $\pi r^2 h$ and the volume of a cone is $\frac{\pi}{3} r^2 H$. They both have the same radius but the volume of the cone is three times as much as that of the cylinder. What is the ratio of the two heights, $H:h$?



…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

10 For what values of x is :

(x + 3)(x - 3) > (x + 4)(x - 3)

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

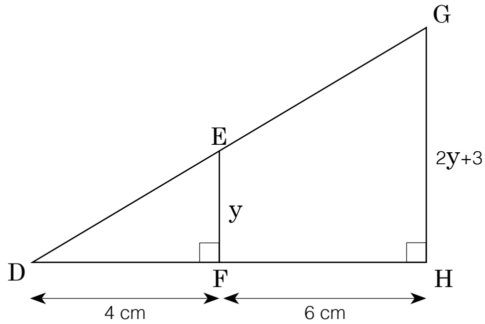
…………………………………………………………………………

…………………………………………………………………………

11 In the diagram below :

DF = 4cm, FH = 6cm, EF = y cm and GH = 2y + 3 cm

Using similar triangles, find the value of y.



…………………………………………………………………………

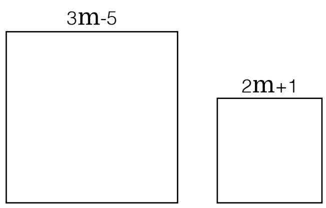
…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

12 Write an expression for the sum of the areas of the two squares :



…………………………………

…………………………………

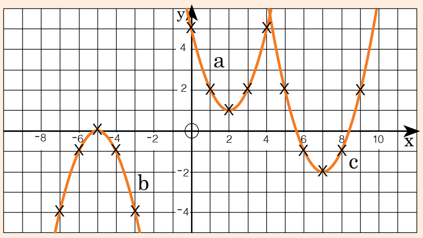
…………………………………………………………………………

…………………………………………………………………………

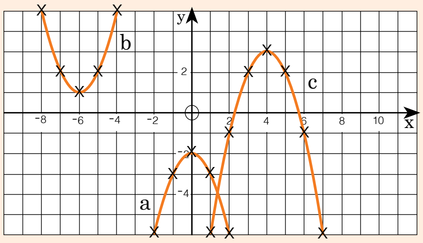
Working Area

Page 109 - Sketching Parabolas 2

A1 a), b), c)



A2 a), b), c)

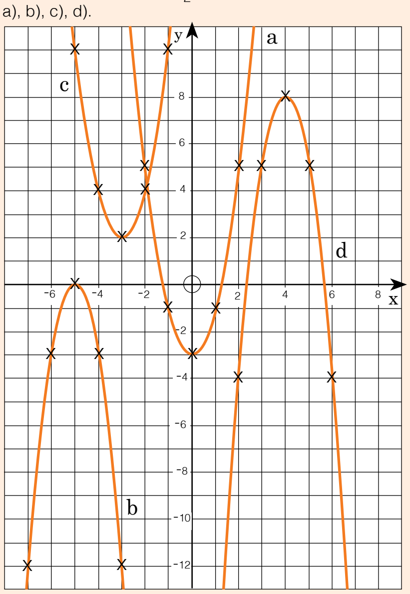


B1 a) $y = (x + 9)^2 - 1$ b) $y = (x + 4)^2 + 2$
c) $y = {}^-x^2 + 2$ d) $y = (x - 3)^2$
e) $y = {}^-(x - 7)^2 + 4$

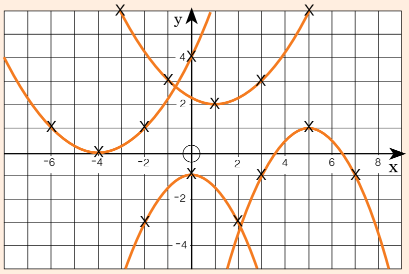
Page 110 - Wide and Narrow Parabolas 1

A1 a) y-column : 8, 2, 0, 2, 8;
pattern : out 1, up 2, out 2, up 8
b) y-column : −12, −3, 0, −3, −12;
pattern : out 1, down 3, out 2, down 12
c) y-column : 1, $\frac{1}{4}$, 0, $\frac{1}{4}$, 1;
pattern : out 1, up $\frac{1}{4}$, out 2, up 1
d) y-column : −2, $-\frac{1}{2}$, 0, $-\frac{1}{2}$, −2;
pattern : out 1, down $\frac{1}{2}$, out 2, down 2

B1 a), b), c), d).



B2 a), b), c), d).



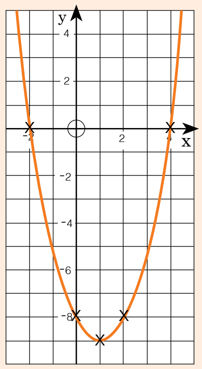
Page 111 - Wide and Narrow Parabolas 2

A1 a) vertex : (−2, −3); pattern : out 1, up 2, out 2, up 8
b) vertex : (0, 1); pattern : out 1, down $\frac{1}{4}$, out 2, down 1, out 3, down $2\frac{1}{4}$, out 4, down 4
c) vertex : (−1, 1);
pattern : out 1, up $\frac{1}{2}$, out 2, up 2, out 3, up $4\frac{1}{2}$

B1 a) vertex : (−2, −4);
pattern : out 1, up $\frac{1}{2}$, out 2, up 2
$y = \frac{1}{2}(x + 2)^2 - 4$
b) vertex : (1, 7);
pattern : out 1, down 2, out 2, down 8
$y = {}^-2(x - 1)^2 + 7$
c) vertex : (3, −7);
pattern : out 1, up 3, out 2, up 12
$y = 3(x - 3)^2 - 7$

Page 112 - Factorised Equations 1

A1 a) $y = 2 \times {}^-4 = {}^-8$ b) $x = {}^-2$ or $x = 4$
c)



d) for vertex $x = 1$,
$y = 3 \times {}^-3 = {}^-9$

B1 y-int : (0, 8) ; x-int (−2, 0) (−4, 0) ;
vertex : (−3, −1)

B2 y-int : (0, −6) ; x-int (2, 0) (−3, 0) ;
vertex : ($-\frac{1}{2}$, $-6\frac{1}{4}$)

B3 a) x-int (−3, 0) (5, 0) ; vertex : (1, −16)
b) x-int (0, 0) (−3, 0) ; vertex : ($-1\frac{1}{2}$, $-2\frac{1}{4}$)

Page 113 - Factorised Equations 2

A1 a) y-int : (0, $-1\frac{1}{2}$) ; x-int (−3, 0) (1, 0) ;
vertex : (−1, −2)
b) y-int : (0, −4) ; x-int (1, 0) (2, 0) ;
vertex : ($1\frac{1}{2}$, $\frac{1}{2}$)
c) y-int : (0, −12) ; x-int (2, 0) (−2, 0) ;
vertex : (0, −12)

B1 a) $y = (x - 3)(x - 2)$
b) y-int (0, 6); x-int (3, 0)(2, 0)
vertex : ($2\frac{1}{2}$, $-\frac{1}{4}$)

B2 a) $y = (x + 1)(x - 3)$
y-int (0, −3); x-int (−1, 0)(3, 0)
vertex : (1, −4)
b) $y = x(x - 2)$
y-int (0, 0); x-int (0, 0)(2, 0)
vertex : (1, −1)
c) $y = (2x + 1)(x - 3)$
y-int (0, −3); x-int ($-\frac{1}{2}$, 0)(3, 0)
vertex : ($1\frac{1}{4}$, $-6\frac{1}{8}$)

Page 114 - Factorised Equations 3

A1 a) $y = (x + 1)(x - 2)$ b) (0, −2)
c) vertex : ($\frac{1}{2}$, $-2\frac{1}{4}$)

A2 a) $y = {}^-(x + 2)(x - 4)$ b) (0, 8)
c) vertex : (1, 9)

A3 a) $y = (x + 3)(x - 1)$ b) (0, −3)
c) vertex : (−1, −4)

A4 a) $y = {}^-(x + 2)(x - 6)$ b) (0, 12)
c) vertex : (2, 16)

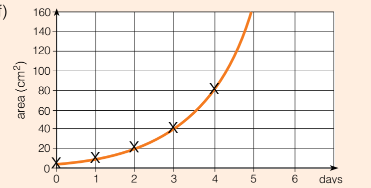
B1 a) $y = a(x + 1)(x - 3)$; $-6 = a \times 3 \times {}^-1$ then $a = 2$
equation : $y = 2(x + 1)(x - 3)$
b) equation : $y = -\frac{1}{3}x(x + 5)$
c) equation : $y = \frac{1}{5}(x + 8)(x - 10)$

Page 115 - Writing Quadratic Equations 1

A1 a) $y = \frac{1}{2}x^2 + 1$ b) $y = \frac{1}{3}(x + 2)(x - 3)$
c) $y = {}^-(x + 2)^2 + 3$ d) $y = 0.3\,x\,(x + 3)$
e) $y = -\frac{2}{3}x^2 + 10$ f) $y = 0.08\,(x - 5)^2$

Page 116 - Exponential Patterns 1

A1 a) day 4, area = 80 cm² ; day 5, area = 160 cm²
b) *'On day 3 the mould covered an area of 40 cm².'*
c) 5×2^n
d) The variable n is in the exponent.
e) When $n = 15$, Area = 5×2^{15} ; Area = 163 840 m².
f)



g) day (−1), area = 2.5 ; day (−2), area = 1.25
or $5 \times 2^{-2} = 1.25$

B1 a) $n = 10$, $t = 59\,049$; rule : $t = 3^n$
b) $n = 10$, $t = 19\,531\,250$; rule : $t = 2 \times 5^n$
c) $n = 10$, $t = 16\,384$; rule : $t = 16 \times 2^n$

B2 Since $16 = 2^4$, the rule becomes $t = 2^4 \times 2^n = 2^{n+4}$

B3 table : x column (top to b) = 1, 2, 3.
y column (top to b) = 20, 80, 320.
work backwards : when $x = 0$, $y = 5$; $y = 5 \times 4^n$.

Page 117 - Exponential Patterns 2

A1 a) 1.35 x \$92 = \$124.20
b) 1.08 x 3000 = 3240 people
c) 0.30 x \$22 000 = \$6600
d) 0.82 x 640 L = 524.8 L

A2 2013 - \$459 000 ; 2014 - \$468 180 ;
2015 - \$477 543.6

B1 a) Year 2 : 1.10 x 5500 = \$6050
Year 3 : 1.10 x 6050 = \$6655
b) Amounts are multiplied by the same number : 1.10
c) $A = 5000 \times 1.10^n$
d) (4) 7320.50 ; (5) 8052.55 ; (6) 8857.81 ;
(7) 9743.59 ; (8) 10717.94. Answer : 8 years

C1 a) Year 2012 : population 720, 2013 : population 864
2014 : population 1036
b) $p = 500 \times 1.2^t$
c) In 2025, $t = 15$, $p = 500 \times 1.2^{15}$
population : 7703 penguins.

Page 118 - Writing Equations Using Technology

A1 a) $y = 1.5x^2 + 1$ b) 151
A2 a) $y = 2.25x + 10.35$ b) 32.85
A3 a) $y = 2.5 \times 2.2^x$ b) 6640.0 (1 dp)
A4 a) $y = 0.5x^2 + 35x + 3$ b) 403
A5 a) $y = 1000 \times 1.1^x$ b) 2593.7 (1 dp)

Pages 119-122 - Practice Exam Questions

A1 $y = -2x^2 + 28x$
Points of intersection (1.169,30) and (12.831,30)
Horizontal distance travelled = 11.662m

2 a) $x^2 + (x+ 2)^2 = 100$
b) $(x + 8)(x - 6) = 0$
$x = -8, 6$
Can't have negative length so x = 6cm

3 $\frac{4}{x} + \frac{x+1}{3} = \frac{12}{3x} + \frac{x(x+1)}{3x} = \frac{12}{3x} + \frac{x^2+x}{3x} = \frac{x^2+x+12}{3x}$

4 $PO = \frac{\sqrt{200}}{2}$ RPT = 85.1° (1 d.p.)

5 $x = \pm\frac{5}{3}\sqrt{h}$

6 58

7 4 m and 7m

8 g = 197

9 9:1

10 $x < 3$

11 $\frac{y}{4} = \frac{2y+3}{10}$
y = 6cm

12 13m² - 26m + 26

13 14x + 20

14 a) 49.6° (1 d.p.) b) 18.8cm (1 d.p.)