A An Infinite Number of Solutions

So far we have solved linear equations with one variable e.g. $2x + 5 = 19$, here the variable is x and the solution to the equation is $x = 7$ because $2 \times 7 + 5 = 19$.

Now we will look at linear equations with 2 variables, like $3x + 2y = 15$. Here the two variables are x and y, that means a solution is a pair of values one x and one y belonging together. The equation $3x + 2y = 15$ has infinite solutions.

For instance : $x = 1,\ y = 6$ because $3 \times \mathbf{1} + 2 \times \mathbf{6} = 15$
$x = 3,\ y = 3$ because $3 \times \mathbf{3} + 2 \times \mathbf{3} = 15$
$x = 6,\ y = {}^-1\frac{1}{2}$ because $3 \times \mathbf{6} + 2 \times {}^-\mathbf{1\frac{1}{2}} = 15$.

For any chosen value of x, you can find a value for y to go with it :
e.g. if $x = 20$, then $3 \times 20 + 2y = 15$ (− 60)
$2y = {}^-45$ (÷ 2)
$y = {}^-22\frac{1}{2}$

So, $x = 20,\ y = {}^-22\frac{1}{2}$ is another solution.

1 Given is an equation with two variables: $4x + y = 9$.

a) Show that $x = 5,\ y = {}^-11$ is one of its solutions.

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b) Is $x = 1,\ y = {}^-13$ part of the solution set?

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c) In one of the solutions $x = 7$. What must be the y-value?

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d) In another solution $y = 7$. What must be the x-value?

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2 Find 2 solutions to this linear equation with two variables : $2x - 5y = 13$. For each solution the value of one of the variables is given.

a) $y = 1$, ...

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Solution : x = ... , y = ...

b) $x = 4$, ...

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Solution : x = ... , y = ...

B Solutions Presented in a Table

Sets of solutions are often shown in a table. We choose a set of values of one variable and calculate the value of the second variable. The solution can be shown as an **ordered pair** (x, y).

Example : Given is the equation $2x + y = 12$.

a) Set up a table of solutions, with x going from ${}^-2$ up to 3.

b) Describe the pattern shown by the set of solutions.

Working : a)

x	$2x + y = 12$	y	solution
⁻2	$^-4 + y = 12$	16	(⁻2, 16)
⁻1	$^-2 + y = 12$	14	(⁻1, 14)
0	$0 + y = 12$	12	(0, 12)
1	$2 + y = 12$	10	(1, 10)
2	$4 + y = 12$	8	(2, 8)
3	$6 + y = 12$	6	(3, 6)

b) I notice that when the x values go up in ones, the y values go down in twos.

1 Given is the equation $4x - y = 2$.

a) Set up a table of solutions with x going from 3 down to ${}^-2$.

x	$4x - y = 2$	y	solution
3	*12 − y = 2*	*10*	*(3, 10)*
2			

b) Describe the pattern shown by the set of solutions.

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2 Given is the equation $x + 2y = 6$.

a) Set up a table of solutions with x going from ${}^-2$ up to 3.

x	$x + 2y = 6$	y	solution

b) Describe the pattern shown by the set of solutions.

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25 Using the Equation

A No Graphs Needed

Many questions about points and lines can be answered without drawing the graph.

Examples :

a) $A = (4, 3)$, $B = (^-4, ^-2)$, $C = (^-4, ^-4)$

Which of these points lie(s) on line $y = \frac{3}{4}x - 1$?

b) Point Q is the point where the line $y = 4$ crosses the line $2x + 3y = 6$. What are the coordinates of Q?

Working :

a) Substitute x and y values into the equation $y = \frac{3}{4}x - 1$
Check whether the equation is true.

Point A : $x = 4$, $y = 3$ $3 \neq \frac{3}{4} \times 4 - 1$ Not true

Point B : $x = {^-4}$, $y = {^-2}$ $^-2 \neq \frac{3}{4} \times {^-4} - 1$ Not true

Point C : $x = {^-4}$, $y = {^-4}$ $^-4 = \frac{3}{4} \times {^-4} - 1$ True

So point C $(^-4, ^-4)$ is on the line.

b) Since Q is on the line $y = 4$, its y-coordinate is 4; $Q = (x, 4)$
Since Q is also on the line $2x + 3y = 6$ then $2x + 3 \times 4 = 6$
Solving $2x + 12 = 6$ gives $x = {^-3}$.
Therefore $Q = (^-3, 4)$.

1 $A = (0, ^-2)$ $B = (8, 0)$ $C = (8, ^-2)$
Which of these points are on the line $x - 4y = 8$?

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2 T is the point where the line $x = {^-2}$ crosses the line $3x + 4y = 0$. What are the coordinates of T?

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3 Point R is on the x-axis and also on the line $y = 3x - 5$
What are the coordinates of R?

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4 $A = (4, 0)$ $B = (4, 1)$ $C = (2, ^-1)$
Which of these points is on line $x - 2y = 4$ as well as line $y = x - 3$?

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B Jobs

1 $E = (2, 0)$ $F = (^-2, 2)$ $G = (4, ^-1)$
One of these points is the intersection of lines $y = 1 - \frac{1}{2}x$ and $x + 4y = 6$. Which point is it?

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2 An IT technician charges \$46 call-out cost and \$84 per hour.
Formula : $C = 46 + 84h$

a) How much does he charge for a call-out job taking 2 hours and 30 minutes?

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b) How much for a 10 minute call-out job?

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c) The technician charged \$95 for a call-out job. How long did the job take?

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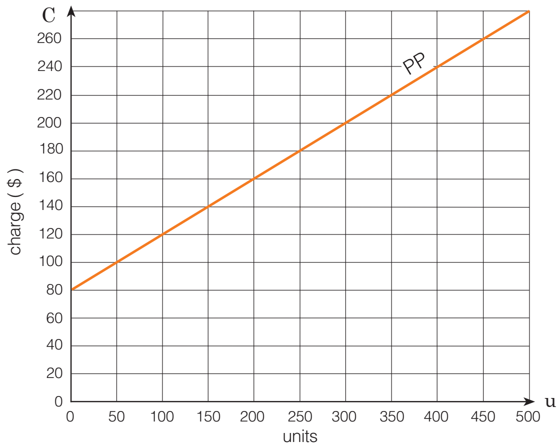
31 Solving Linear Problems 2

A The Cost of Electricity

1 In our town we have the choice between two electricity providers : *Power People* (PP) and *Electric Company* (EC).
Both companies have a fixed line charge and a cost per unit.
The graph on the right shows the monthly charges of PP.



a) EC charges $40 fixed line charge plus 60 cents per unit.
This can be described by the equation $C = 0.6u + 40$.
Draw the graph of EC's monthly charges on the grid.

....

b) Which company has a higher charge per unit? ....

c) Write an equation for the charges of PP.

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d) The Crawford family uses 350 units per month. How much could they save by selecting the best power company?

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2 A third power company called *ZipZap* (ZZ) joins the market. They too have a fixed line charge and a cost per unit.
ZipZap aims to undercut both PP and EC for households using at most 400 units per month.

a) Draw a line showing the possible charges of ZZ.

b) Describe the charge of ZZ in words. ....

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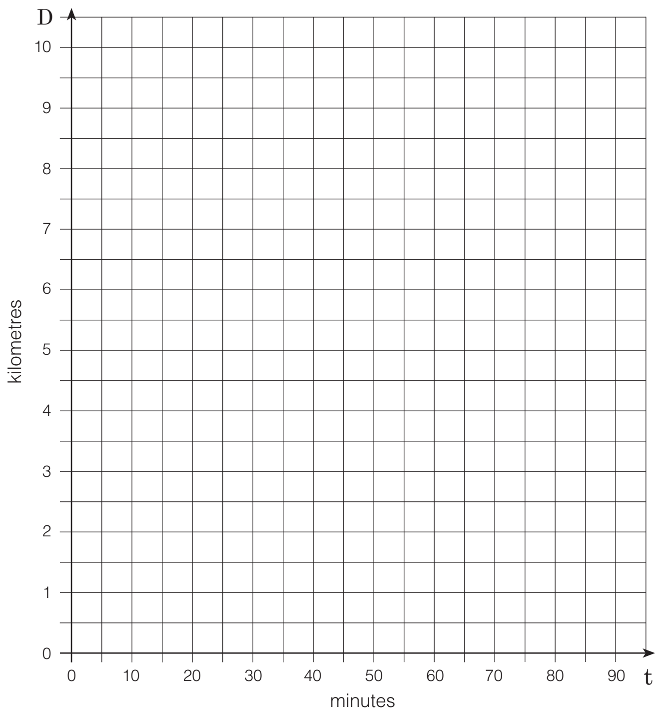
B Training Session

1 Amy, Ben and Chloe are training for a cross-country race. Section A is 6 km long on the flat, section B is 4 km on hilly terrain.

- Amy takes 40 min for section A, 50 min for section B.
- Ben starts 5 min after Amy and finishes section A 5 min before her. He runs section B at a speed of 6 km/h.
- Chloe starts with Amy, goes as fast as Ben on section A, but slows down to 4 km/h on section B.



a) Draw lines to show the situation as it develops.

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b) Calculate Amy's speed on section B.

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c) Calculate Chloe's average speed for the entire race.

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d) Give times where people pass each other. ....

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39 Linear Programming 1

A The Hot Baker

1 Every day *'The Hot Baker'* bakes white buns and rye buns.

a) The maximum number buns they bake in a day is 90. Write an inequation for this information, using w for the number of white buns and r for the number of rye buns baked in a day.

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b) The demand for rye buns is at least half the number of white buns. Write another inequation.

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c) Also known is that w and r are whole numbers. Why?

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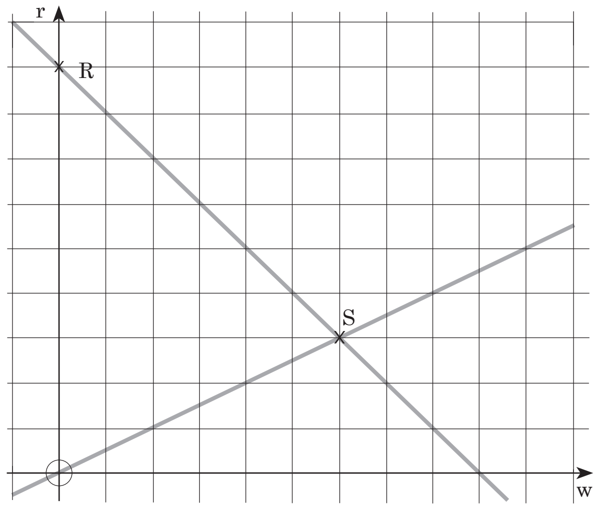

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d) The region where both inequations are true at the same time can be shown on a graph. The boundary lines of the region are shown in the graph.

i) Write an equation on each line.

ii) Work out the coordinates of R and S. ....

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iii) Shade the region where both inequations are true simultaneously. ....

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e) i) The apprentice baker wants to bake 50 white and 20 rye buns. Is this a good idea? Say why.

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ii) Can you recommend a better number of rye buns to go with 50 white buns? Explain. ....

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f) Suppose the profit on white buns is higher than the profit on rye buns. How many of each should they bake? Explain.

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