

10 Mental x and ÷



Whole Numbers

A Counting Zeros

The standard strategy for multiplying numbers with a lot of zeros is **rearranging**.

$$\begin{aligned} \text{Example : } 200 \times 30 &= 2 \times 100 \times 3 \times 10 = 2 \times 3 \times 100 \times 10 \\ &= 6 \times 1000 = 6000 \end{aligned}$$

1 Rearrange and calculate.

a) $400 \times 600 =$

b) $20 \times 15000 =$

Mental short-cut rule when multiplying large numbers :

Multiply the non-zero digits, write down the result and attach the total number of zeros present in the multiplication.

Examples : Calculate a) 60×500 b) 120×30

a) $\boxed{6}0 \times \boxed{5}00 = \boxed{30}000$ b) $\boxed{12}0 \times \boxed{3}0 = \boxed{36}00$

Answers : a) 30 000 b) 3600

2 Calculate

a) $80 \times 9000 =$

b) $600 \times 70 =$

c) $1200 \times 300 =$

d) $400 \times 110 =$

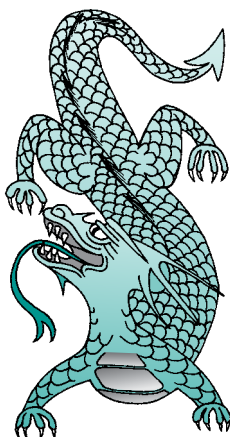
e) $200 \times 46 =$

f) $60 \times 1200 =$

g) $3000 \times 25 =$

h) $780 \times 200 =$

i) $350 \times 40 =$



3 Rory found the answer to $4 \times 8 \times 7 \times 5$ by rearranging.

$$4 \times 8 \times 7 \times 5 = 4 \times 5 \times 8 \times 7 = 20 \times 56 = 1120$$

Use Rory's way to calculate :

a) $6 \times 9 \times 5 \times 3 =$

b) $15 \times 3 \times 4 \times 7 =$

B Turn It Around

Division is the **inverse** of multiplication.

Therefore we change the division into a multiplication and solve.

Example : Calculate $\frac{18\,000}{300}$

Working : Change into $300 \times \dots = 18\,000$

We need 6 to make 18; we also need an extra zero.

$$300 \times \underline{60} = 18\,000$$

Answer : $\frac{18\,000}{300} = 60$

1 Complete : Since $800 \times \dots = 240\,000$,
then $\frac{240\,000}{800} =$

2 Mentally change into a multiplication, write the answer.

a) $\frac{63\,000}{900} =$ b) $\frac{160\,000}{40} =$

c) $\frac{40\,000}{800} =$ d) $\frac{240}{12} =$

e) $\frac{66\,000}{110} =$ f) $\frac{3200}{1600} =$

Divisions can be made simpler by **halving both numbers**.

Examples : Calculate

a) $\frac{68}{4}$ b) $\frac{120}{8}$

$$\frac{68}{4}$$

$$\frac{120}{8}$$

Working :

a) $\frac{68}{4} = \frac{34}{2} = 17$

b) $\frac{120}{8} = \frac{60}{4} = \frac{30}{2} = 15$

3 Calculate

a) $\frac{64}{4} = \dots = \dots$ b) $\frac{124}{4} = \dots = \dots$

c) $\frac{1024}{4} = \dots = \dots$ d) $\frac{90}{6} = \dots = \dots$

e) $\frac{208}{8} =$

f) $\frac{1360}{8} =$

g) $\frac{224}{16} =$

h) $\frac{288}{12} =$



Number Facts

Integer Arithmetic 23

A The Value of a Hand

We play a card game in which the spades and clubs (♠ ♣) have a positive value, hearts and diamonds (♥ ♦) have a negative value. Picture cards are all worth 10, Aces 20.

Examples : Write a number sentence for the value of these hands of cards.

a)



b)



Answers : a) $-20 + 6 = -14$

b) $3 + 4 + 8 = 7$

1 Write number sentences for the values of the hands.

a)



b)



c)



d)



e)



B Removing Cards

Examples : This hand is worth 7.

Write a number sentence and calculate the value after we remove . . .

a) the black 3 b) the red 4



Working : We have 7, we take away the card's value.

Calculate by imagining the hand without the card.

Answers : a) $7 - 3 = 4$ b) $7 - 4 = 3$

1



This hand is worth 1.

Write a number sentence for the value after we remove one card.

a) remove the black 8 $1 - 8 =$

b) remove the red 5 $1 -$

c) remove the red 2 $1 -$

2 This hand is worth -5.

Write a number sentence for the value after we remove one card.



a) remove the red jack $-5 - 10 =$

b) remove the black 7 $-5 -$

c) remove the red 6 $-5 -$

d) remove the black 4 $-5 -$

C Think of Cards

When we add a positive number (black card) to a hand, the total value goes up. If we remove a positive number, the total value goes down. When we add a negative number (red card) to a hand, the total value goes down. If we remove a negative number, the total value goes up.

1 Complete these.

a) $6 + -4 =$ b) $8 + -10 =$ c) $-3 + 7 =$ d) $-9 + 4 =$

e) $-4 + -5 =$ f) $7 + -7 =$ g) $10 + -15 =$ h) $-5 + -8 =$

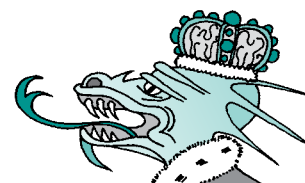
2a) $4 - 8 =$ b) $10 - -6 =$ c) $-2 - 4 =$ d) $3 - 9 =$

e) $-1 - 10 =$ f) $3 - -4 =$ g) $-8 - -1 =$ h) $2 - 2 =$

3 Two red kings equal -20 points. Complete the family of facts.

Since $2 \times -10 = -20$, then $\times$ = -20

also $-20 \div$ = and $-20 \div$ =



30 More Strategies + and -



Decimals

A Adding On

Adding on is another useful strategy when subtracting decimals.

Example : Calculate $28.25 - 14.8$

Working : Start with 14.8. What needs to be added to make 28.25?
 $14.8 + 0.2 + 13 + 0.25 = 28.25$

Answer : $28.25 - 14.8 = 13.45$

1 Complete

a) $42.3 + \dots = 54.08$

Therefore $54.08 - 42.3 = \dots$

b) $5.87 + \dots = 9.3$

Therefore $9.3 - 5.87 = \dots$

c) $2.795 + \dots = 10.32$

Therefore $10.32 - 2.795 = \dots$

2 Work these out mentally.

a) $8.26 - 3.7 = \dots$

b) $29.4 - 16.85 = \dots$

c) $41.05 - 34.975 = \dots$

C Match Making

Ovals hold questions, circles hold answers.

- ◆ Shade the six ovals in different colours.
- ◆ Find a matching answer to each question.
- ◆ Shade the answer the same colour as the question.

$43.4 - 18.6$	2.68	25.6
$20.025 + 4.575$	24.8	2.82
$73.2 - 14.84$	24.08	62
$9 - 6.18$	2.6	24.6
$4.24 - 1.56$	58.36	60.2
$31.42 + 28.78$		

B Adding Lots of Decimals

1 You may add numbers in any order.
Make it easier by choosing compatible numbers.

a) $9.3 + 5.6 + 13.4 + 2.7 = \dots$

b) $3.72 + 6.43 + 2.28 + 8.57 = \dots$

c) $12.45 + 9.4 + 11.3 + 8.6 + 4.25 = \dots$

2 Adding almost equal numbers can be made easier by multiplying and adjusting.

a) $2.9 + 2.7 + 2.8 + 3.4 + 3.5$
 $= (5 \times 3) - 0.1 - \dots$

b) $10.25 + 10.85 + 9.8 + 10.05 + 9.9 = \dots$

D Magic Squares

1 Complete this magic square.
All rows, columns and diagonals must add to the number 7.

1			1.3
	1.5		1.8
1.7			
	1.2	1.1	2.5

2 Here is another magic square.
This time the magic number is 4.

		1.35	0.25
0.45			1.65
	1.25		
1.75	0.65		0.55





Fractions

A Proportions

Proportion is another word for fraction.

Examples :

Of twenty cars tested, three failed their warrant of fitness.

- What proportion of the cars failed?
- What percentage of the cars failed?

Working :

- Write 3 out of 20 as a fraction ; Answer : $\frac{3}{20}$
- $\frac{3}{20} = \frac{15}{100}$; Answer : 15%

1 *Nine out of ten nutritionists recommend Weetbix.*

- What proportion of nutritionists recommend Weetbix?

.....

- What is that as a percentage?

2 *Out of a class of 25 pupils, three are absent.*

- What proportion of the class is absent?

- What percentage is absent?

3 *Thirteen out of fifty apple trees were damaged by the storm.* Rewrite this statement with a percentage.

..... percent of the apple trees were damaged by the storm.

4 Write Aaron's test results as percentages.
Which was his best test?

Test A : 39 out of 50 %

Test B : 16 out of 20 %

Test C : 75 out of 100 %

Aaron's best test was

5 Sharon did a test with 80 questions.
She answered 67 questions correctly.

- What proportion of the questions did Sharon answer correctly?

- Write this proportion as a decimal.

- Sharon reckons she had more than 80% correct.

Do you agree?

Using Percentages 49

B Save the Kiwi

Example : A shop offers 25% discount on a pair of shoes which have a price tag of \$89. What is the discount in dollars?

Working : 25% of 89 = $\frac{1}{4}$ of 89

Without a calculator : $\frac{1}{2}$ of 89 = 44.50 ; $\frac{1}{2}$ of 44.50 = 22.25

With calculator : $\boxed{89} \boxed{\div} \boxed{4} \boxed{=} 22.25$ Answer : \$22.25

1 Show how you calculate these mentally.

- 10% of \$4.00

.....

- $66\frac{2}{3}\%$ of 300 g

.....

- 40% of 600 ml

.....

- 25% of 84 kg

.....

2 You may use a calculator for these.

- 30% of \$148

.....

- 75% of \$18.50

.....

- 2% of 52.6 km

.....

- 60% of 0.39 L

.....

3 Forty percent of the sixty Year 8 students at Woolston Intermediate walk to school. How many walk to school?

.....

4 90% of kiwi chicks hatched in unprotected areas are killed by dogs and ferrets. If 200 chicks are expected to hatch in unprotected areas next year, how many are expected to be killed?

.....

54 Number Patterns



Algebra

A Discover the Rules

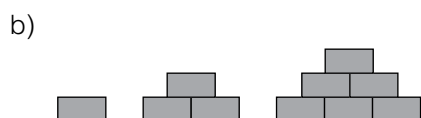
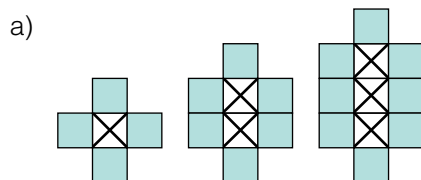
1 Study these sequences, each has their own special rule. When you discover the rule, continue the sequence with three more numbers.

- 5, 11, 17, 23, 29,,,
- 60, 57, 54, 51, 48,,,
- 1, 2, 4, 8, 16,,,
- 7, 6, 9, 8, 11,,,

2 Fill in the missing numbers in each sequence.

- 120, 60, 30,, 7.5,, 1.875
- 1, 3, 9,, 81,, 729
- 7, 1, -5,, -17,, -29
- 1, 4, 9,, 25,, 49

3 Draw the next diagram.



4 For each sequence, find the next two numbers and explain how you did it.

- 50 000, 5000, 500, 50,,
.....
.....
.....
- 1, 2, 4, 7, 11,,
.....
.....
.....

B Follow the Rules

The numbers in a sequence are called **terms**.

For instance in the sequence 6, 1, -4, -9, ... the first term is 6, the second term is 1, etc.

The rule for the sequence is 'the first term is 6, each term is 5 less than the previous term.'

1 Use these rules to write down 5 terms of each sequence.

- The first term is 8, each term is 3 less than the previous term.

.....,,,,

- The first term is 64, each term is half the previous term.

.....,,,,

- The first term is -12, each term is 5 more than the previous term.

.....,,,,

2 The number of rabbits on a farm doubles every month, the first month there were ten.

- Complete the table for this rule :

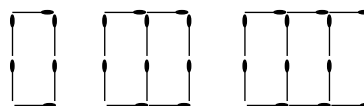
months	1	2	3	4	5	6
rabbits	10					

- How many rabbits will there be after a year?

.....

3 Matches are used to form a string of rectangles.

- Draw a string with 4 rectangles.



- Complete this rule :

The first rectangle in a string needs matches, each following rectangle needs extra matches.

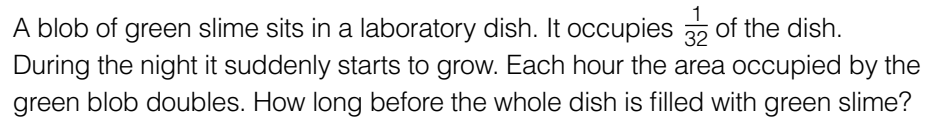
- How many matches are needed for a string of

10 rectangles?

.....



1



1

(Hint : Working backwards you could call yourself generation 0, your parents generation 1 and your grandparents generation 2 and so on.)



72 Rates



Measurement

A Getting Started

Things like cost, speed and rainfall are **rates**.

The unit for a rate contains the word '*per*'.

For instance : The unit for cost is dollars *per* item.

The unit for speed is km *per* hour.

The unit for rainfall is mm *per* year.

1 Match the rate with the unit.

Rate	Unit
speed	kilowatts per hour
growth rate	kilometres per hour
fuel consumption	kilometres per litre
paint coverage	dollars per pound
electricity use	centimetres per year
exchange rate	square metres per litre

2 Think of a possible unit for these rates.

- weight loss per
- heart rate per
- pay rate per
- mail sorting per

3 Jason gets paid \$5.50 per hour to baby-sit his younger sister. How much does he get paid for $3\frac{1}{2}$ hours of baby-sitting?

4 On a paint tin is printed 1 L covers 15 square metres. How many square metres can be covered with 2.5 L?

5 Our car used a full tank of petrol (40 L) to drive 520 km from Hamilton to Wellington. Calculate our car's fuel consumption in kilometres per litre.

6a) Mr Watson exchanged 250 NZ dollars for 17500 yen. At this rate, how many yen do you get for one NZ dollar?

b) How many yen does Lucy Watson get for \$40?

B Speed

Example :

A tourist bus takes 6 hours to travel from Christchurch to Dunedin, which is a distance of 372 km.

- What is the average speed of the bus?
- At this speed, how far can the bus be expected to travel in 15 minutes?

Working :

- 372 km in 6 hours, that is $372 \div 6 = 62$ km in 1 hour.
Speed = 62 km/hr
- 15 minutes is $\frac{1}{4}$ of an hour.
distance travelled = $\frac{1}{4}$ of 62 km = 15.5 km.

1 A plane takes 5 hours to fly 4620 km.

- What is the speed of the plane?
- At this speed, how far does the plane fly in 20 minutes.

2 It took Rewi half an hour to bike 8 km. What was Rewi's speed in km per hour?

3 At an average speed of 100 km/hr, what distance would be covered in . . .

- 15 minutes?
- 6 minutes?

4 Driving at an average speed of 80 km/hr how long will it take us to drive 520 km from Hamilton to Wellington?

5 Kara took 20 minutes to walk the 2 km to Emma's house.

- If Kara kept walking at the same speed, how far would she have walked in 1 hour?

..... km

- Kara ran back home, covering the 2 km in 15 mins. What was Kara's speed on the way home?

..... km/hr

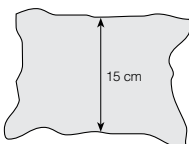




Geometry

A Paper Folding

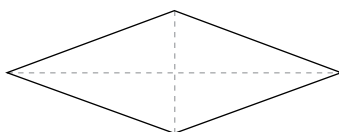
For this exercise you need 6 pieces of paper. The pieces must not have straight edges. For each question you start with a fresh piece of paper.



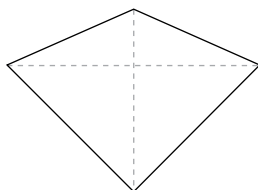
When finished, open up the paper and trace with pen the folds which show the required shape.

- 1 Fold a piece of paper to make an angle of 45° .
- 2 Fold a piece of paper to make a square.

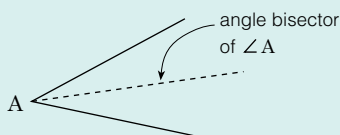
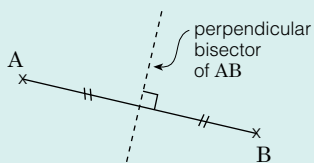
- 3 Fold a piece of paper to make a rhombus.



- 4 Fold a piece of paper to make a kite.

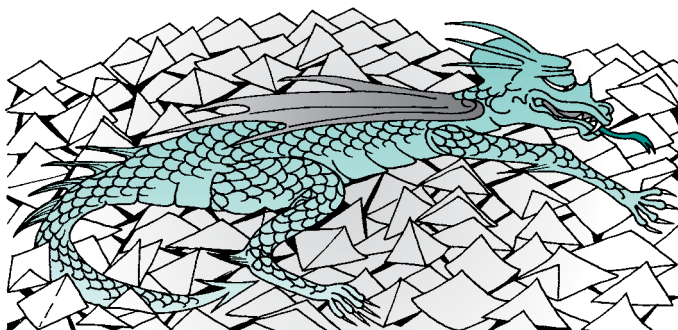


The **perpendicular bisector** of a line segment AB , is a line which cuts AB in half and is perpendicular to AB .



An **angle bisector** is a line which cuts an angle in half.

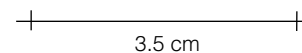
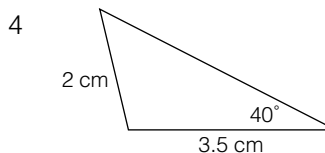
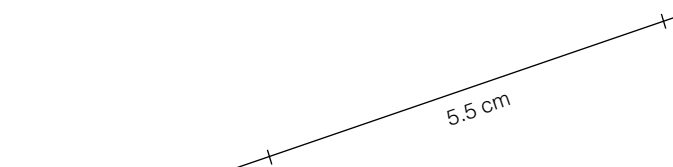
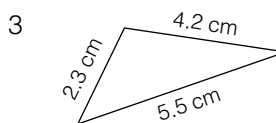
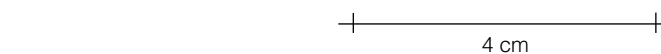
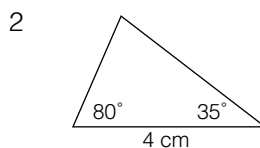
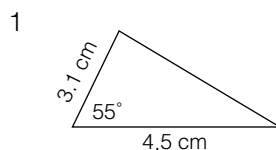
- 5 Mark two points A and B on a piece of paper. Make a fold showing the perpendicular bisector of AB .
- 6 Draw an angle on a piece of paper. Make a fold showing the angle bisector.



Constructions 91

B Compass, Ruler and Protractor

Below are four sketches of triangles. Use your drawing instruments to draw exact copies of each of the triangles.

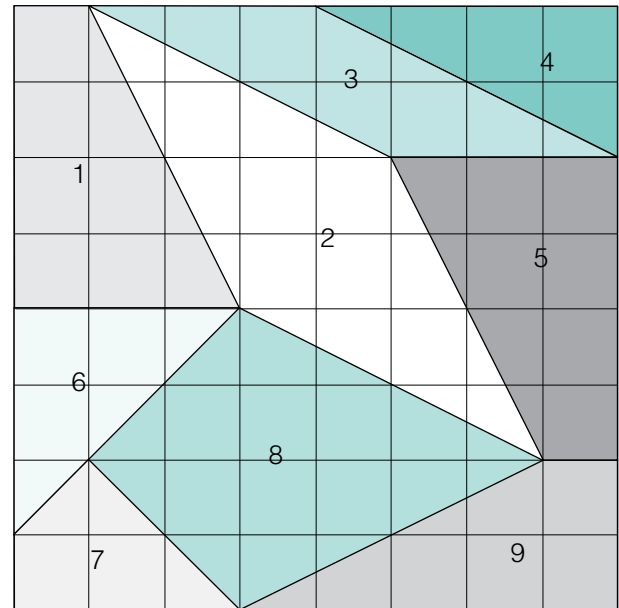




A

Polygons

- 1) Nine polygons are painted on a square tile. Study the shapes.
- ◆ Are they all different or are some of the shapes congruent? Write about each polygon. For instance :
 - ◆ What is its most precise name?
 - ◆ Work out its area and perimeter
 - ◆ Is it a symmetrical shape?



**A New Zealand Households**

- 1 Garth has used a computer spreadsheet to make this pie graph. It shows how the average New Zealand household uses electricity. The pie is drawn in 3D style, which is not always a good idea because it tends to make some slices look bigger than they ought to.

- a) Which categories in the graph look bigger than they should?

Explain your choice.

.....

.....

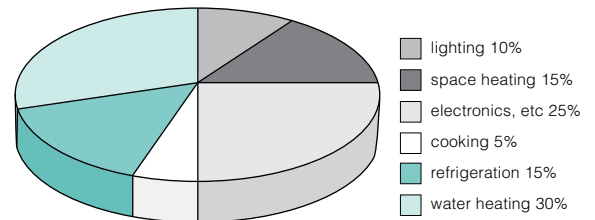
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Electricity Use in the Average NZ Household



- b) Redraw the pie graph in this circle, representing each category fairly. Hint : First calculate 5% of 360 degrees and then use that to calculate all the angles at the centre of the pie.

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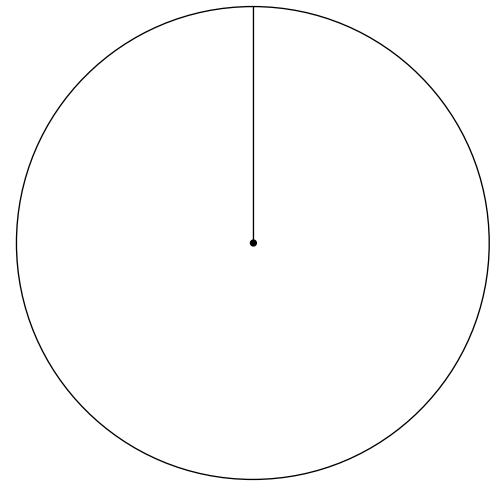
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- 2 In 2016 households spent on average \$220 per week on food. Money was spent in five food subgroups as follows : Fruit and vegetables \$24, meat, poultry and fish \$30, grocery food \$92, beverages (non-alcoholic) \$12, eating out and take-a-ways \$62 (source : Statistics New Zealand).

- a) Garth decides to make a strip graph to illustrate this information and he makes the strip 110 mm long. Finish the graph.

Average Weekly Food Expenses per Household



- b) Do you think that the above graph based on data from 2016 is still useful? Explain your opinion.

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